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ABSTRACT

Intended to provide guidance for secondary school mathematics teachers who desire to maximize the reading abilities of their students and thus maximize students' performance, this report attempts to make the secondary school teacher aware of some of the difficulties encountered in reading mathematics and offers some techniques, activities, and strategies to overcome these difficulties. The content is divided into five categories or factors that contribute to problems in reading mathematics: the language of mathematics, mathematical vocabulary and symbols, word problems, mathematical reading skills, and readability of mathematics textbooks. Each section includes a discussion of the reading problems, suggestions for overcoming difficulties in reading mathematics, and an annotated list of resources. (HOD)

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READING IN SECONDARY MATHEMATICS:
PROBLEMS, SUGGESTIONS, SOURCES

Education 564
August 23, 1984
Cherlyn Hall

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CHAPTER I

INTRODUCTION

Secondary mathematics teachers often face classes in which the students' reading levels span several grades or in which the students lack the basic reading skills required for mathematics. Even when a student has developed general reading skills during his elementary years, there often is not a transfer of those skills to reading mathematics materials at the secondary level as the materials become increasingly complex and abstract. To learn any subject matter the student must develop competence in the language of that subject. Hence, there is a close relationship between effective reading in mathematics and performance in mathematics.¹ The high school mathematics teacher therefore must attempt to remedy this situation. The mathematics teacher must teach reading to the extent required to help her students develop the reading skills needed to understand the subject of mathematics.

Many factors cause high school students to have trouble comprehending mathematics materials. Among the most obvious factors are: a student's interest and motivational level, a student's reading level in ordinary English, and a student's background of experience and ideas. Researchers attribute reading difficulties in mathematics to several factors. First,

most researchers agree that the language of mathematics often presents unique problems to students because of its conciseness, abstractness, highly technical vocabulary and symbols, and complex relationships.² Second, because of the nature of the language of mathematics, students must approach reading mathematics differently than reading a story in literature, for example.³ Third, everyday words often take on different meanings in the language of mathematics. Finally, different skills are needed to read mathematics materials than other subjects.⁴

In recent years, educators have debated the propriety of reading instruction in the content areas. Some educators believe that reading instruction should take place only in the elementary grades.⁵ Admittedly, that is where learning to read begins, but as a student progresses in school reading becomes the tool by which he learns practically everything else. Reading is a process, rather than a classroom subject. According to Hollander, 85 percent of the learning which takes place in the secondary school is based on reading.⁶ Other educators believe that only students with general reading problems need additional instruction.⁷ As a student progresses in school and the reading process changes, however, reading skills need to be sharpened and expanded. Students must begin reading different kinds of materials for various purposes, and they need guidance and practice in locating, comprehending and procuring information contained in the various curriculums.⁸ Many students can "read" fluently, yet comprehend little of what they encounter. Reading is more than just pronouncing words---it involves extracting

meaning. Secondary teachers are sometimes reluctant to teach reading in their classes because they feel it is not their responsibility or because they feel they do not have the skills and specialized techniques needed for reading instruction.⁹ Reading in the content fields does require special skills. The teacher with expertise in the specific subject is the one best situated to teach those skills.¹⁰

The purpose of this paper is to provide guidance for the secondary mathematics teacher who desires to maximize the reading abilities of her students and thus maximize the students' performance. This paper attempts to make the secondary teacher aware of some of the difficulties encountered in reading mathematics and offers some strategies to overcome these difficulties. The factors contributing to problems in reading mathematics are divided into five categories: 1) The language of mathematics, 2) Mathematical vocabulary and symbols, 3) Word problems, 4) Mathematical reading skills and 5) Readability of mathematics textbooks. Included in each category is a discussion of the reading problem, suggestions for overcoming difficulties in reading mathematics, and an annotated listing of resources recommended by the author for both new and experienced secondary mathematics teachers to use for teaching reading in the mathematics classroom. These categories are not an exhaustive analysis of all aspects of reading difficulties in mathematics. For example, factors such as a student's intelligence, interest, motivational level, and background influence reading in mathematics. The categories often interrelate and overlap. The

paper does, however, provide a sampling of techniques, activities, strategies and resources of some of the major components of reading difficulties in mathematics.

FOOTNOTES

Many of the sources referred to in this paper are cited in more than one chapter. In those cases, the full citation is given in the first chapter in which the source appears. Thereafter, the following format is used: Doe, title of book or article, p. 30. The reader may then refer to the Bibliography for the full citation of the source written by Doe.

¹Lewis R. Aiken, Language Factors In Learning Mathematics (Columbus, Ohio: ERIC Document Reproduction Service, ED 068 340, 1972), p. 18; Robert C. Aukerman, "Reading In The Mathematics Classroom," Reading In The Secondary School Classroom (New York: McGraw-Hill Book Company, 1972), p. 189; Calhoun C. Collier and Lois C. Redmond, "Are You Teaching Kids To Read Mathematics?" The Reading Teacher Vol. 27 No. 8 (May, 1974): 804; Texas Education Agency, Learning Partners: Reading and Mathematics (ERIC Document Reproduction Service, ED 179 422, 1979), p. 1; Stephen Krulik, "To Read or Not To Read, That Is The Question!" The Mathematics Teacher Vol. 73 (April, 1980): 248; Susan M. Pirne, Teaching Reading In The Mathematics Class (ERIC Document Reproduction Service, ED 228 618, 1983), p. 38.

²Collier and Redmond, "Are You Teaching Kids To Read Mathematics?", p. 805; Florida State Department of Education, Reading The Language of Mathematics (ERIC Document Reproduction Service, ED 134 972, 1975), p. iii; Sheila K. Hollander, Reading The Special Language of Mathematics (ERIC Document Reproduction Service, ED 137 726, 1977), p. 5; Fred Lees, "Mathematics and Reading," Journal of Reading Vol. 19 No. 8 (May, 1976): 621; James D. Riley and Andrew B. Pachtman, "Reading Mathematical Word Problems: Telling Them What To Do Is Not Telling Them How To Do It," Journal of Reading Vol. 21 No. 6 (March, 1978): 531; Vicki J. Schell, "Learning Partners: Reading and Mathematics," The Reading Teacher Vol. 35 (February, 1982): 544; Cyrus F. Smith and Henry S. Kepner, Jr., Reading In The Mathematics Classroom (ERIC Document Reproduction Service, ED 207 836, 1981), p. 9; Horst G. Taschow, Reading In Mathematics (ERIC Document Reproduction Service, ED 046 670, 1970), p. 1; Texas Education Agency, Learning Partners: Reading and Mathematics, p. 1.

³Aukerman, "Reading In The Mathematics Classroom," p. 189; Lawrence E. Hafner, "Reading Mathematics. Strategies For Improving The Reading of Mathematics," Developmental Reading In Middle and Secondary Schools. Foundations, Strategies and Skills For Teaching (New York: MacMillan Publishing Company, 1977), p. 258; Hollander, Reading In The Special Language of Mathematics, p. 5; Ellen Lamar Thomas and Alan H. Robinson, "Mathematics," Improving Reading In Every Class. A Sourcebook For Teachers

(Boston: Allyn and Bacon, Inc., 1972), p. 277; Krulik, "To Read Or Not To Read, That Is The Question!", p. 248.

⁴Collier and Redmond, "Are You Teaching Kids To Read Mathematics?", p. 805; Ann B. Dolgin, "Improvement of Mathematical Learning Through Reading Instruction," The High School Journal Vol. 61 (November, 1977): p. 59; David L. Shephard, "Applying Reading Skills To Mathematics," Comprehensive High School Reading Methods. Third Edition (Columbus, Ohio: Charles E. Merrill Publishing Company, 1982), p. 307; Thomas and Robinson, "Mathematics," p. 277.

⁵Adam S. Toback, "Secondary Teachers Can Teach Reading," The Clearing House Vol. 49 (1976): 249.

⁶Hollander, Reading The Special Language of Mathematics, p. 1.

⁷Toback, "Secondary Teachers Can Teach Reading," p. 249.

⁸Hollander, Reading The Special Language of Mathematics, p. 4; David Moore, John E. Readence and Robert J. Rickelman, "An Historical Exploration of Content Area Reading Instruction," Reading Research Quarterly Vol. 18 No. 4 (Summer, 1983), p. 420; Merritt Edward Wadleigh, A History of Reading Programs Involving Instruction In The Content Areas, 1966-1981 (ERIC Document Reproduction Service, ED 225 145, 1982, p. 17.

⁹Hollander, Reading In The Special Language of Mathematics, p. 2; William S. Palmer, Teaching Reading In The Content Areas: Three Pitfalls, Three Possibilities (ERIC Document Reproduction Service, ED 088 022, 1974), p. 1; Toback, "Secondary Teachers Can Teach Reading," p. 248.

¹⁰Dolgin, "Improvement of Mathematical Learning Through Reading Instruction," p. 59; Hollander, Reading In The Special Language of Mathematics, p. 15; Palmer, Teaching Reading In The Content Areas: Three Pitfalls, Three Possibilities, p. 2.

CHAPTER II

THE LANGUAGE OF MATHEMATICS

A. Discussion of the Reading Problem

Researchers contend that the nature of the language of mathematics makes it difficult to read for many students. For example, Schell contends that "mathematics is the most difficult content area material to read, with more concepts per word, per sentence, and per paragraph than any other area"¹, and Manzo states "there are more computational morons and math illiterates than there are people who cannot read."² The language of mathematics is precise, abstract, and composed of highly technical vocabulary and symbols which often present complex relationships. Shephard states that mathematics comprises two languages: 1) the "language of words" which includes the technical vocabulary and the peculiar usage of common words; and 2) the "language of symbols" which represents words or phrases indicating relationship or process.³ Lerch elaborates further by saying that in mathematics the words are used in explanations, descriptions, directions and word problems.⁴ Mathematical symbols are used in much the same way in illustrations, examples and descriptions. The mathematics student must be able to read these two languages and translate from one to the other.

The language of mathematics differs significantly from ordinary English. Several differences in the two languages

have been noted by the Texas Education Agency.⁵ First, ordinary English relies on a standard set of twenty-six alphabetic symbols; whereas, mathematics uses those symbols plus many nonalphabetic, mathematical symbols. Second, the sentence structure in mathematics differs from that of standard English. For example, Schell models a mathematics sentence in the following manner: a symbol names things, followed by a symbol that names a relationship, followed by a symbol that names things.⁶ This structure is illustrated by the following mathematics sentence $8x + 2 = 2x + 4$, in which " $8x + 2$ " and " $2x + 4$ " are symbols naming things and "=" names a relationship. Third, in mathematics the student does not always read from left to right and down the page. Fourth, Hafner attributes the difficulties in reading the language of mathematics, in part, to the fact that mathematicians write for one another instead of for students.⁷ As the secondary student encounters the special language of mathematics new demands are placed on his reading power. He must draw meaning from often dense analytical passages in a language with which he is often unfamiliar.

B. Strategies, Activities and Exercises for Overcoming Mathematics Reading Difficulties

There are several ways in which the secondary mathematics teacher can help her students extract meaning from the language of mathematics. First, she can provide the students with insight into the structure of the mathematics language by reading a sequence from an English literature text and then reading a portion of a mathematics text thus making the student

aware of the ways in which the language of mathematics differs from the language of literature. She should point out that the language of mathematics is comprised not only of words, but also of specialized symbols, and that in mathematics words sometimes have different meanings than in common usage. For example, the words face, natural, set, rational, product, odd, volume, mean and power have different meanings in mathematics than in everyday usage. The teacher could discuss both meanings of these words when they are encountered in mathematics, and have the students keep a dictionary of these words which includes both meanings and sentences which use the words in a mathematical and nonmathematical context.⁸ She should also discuss the structure of the mathematical sentence and emphasize that every word and symbol is crucial to the mathematical sentence. She might point out the similarities between mathematical statements and grammatical statements. Numerals, variables, geometric points and many other mathematical elements work the same way as nouns. There is a strong similarity between verbs and symbols of relation such as equality ($=$), size ($>$), congruence ($\cong$), and membership ($\in$). Punctuation, in mathematics as well as in English, plays an important role in removing ambiguity. Parentheses, for example, tell the mathematics student which operation to perform first.⁹ In addition to helping the student understand the structure of mathematical sentences, the teacher might also help the student realize the flexibility of the order of the sentence in mathematics. She might ask the students to write mathematical sentences in as many different forms as

possible.¹⁰ For example, the sentence $396 = 300+90+6$ can be written as $896 = (8 \times 100) + (9 \times 10) + (6 \times 1)$, or as eight hundred ninety-six equals $800+90+6$, or as eight hundred plus ninty plus six = 896. Finally, the teacher should point out the differences between the symbolism in the current mathematics text and the mathematics text the students used the previous year because the symbolism may vary from one mathematics series to another.

Second, the mathematics teacher can point out that because the language of mathematics is different the student must abandon the traditional method of rapidly reading the material once straight through and instead read slowly and carefully. To encourage the student to read mathematical material in a slow, precise manner Krulik suggests that the teacher mimeograph several pages from the textbook, mount them on a piece of poster-board, and cut them into a jigsaw puzzle.¹¹ The students then are given the puzzle pieces in an envelope and are instructed to reassemble the printed page. This activity encourages the student to read carefully and slowly to find the meaning of the full sentences from the fragments given to him.

Finally, the teacher can note that the eyes move different directions when reading mathematics--not just the usual left-right and up-down movements.¹² For example, when reading a graph or table the student may need to read left to right, right to left, top to bottom, bottom to top or horizontally or vertically simultaneously. Also, a simple expression such as $\frac{7}{2}x - \frac{1}{2} = \frac{3}{4}x$ is read in a variety of directions. The teacher can provide examples and techniques illustrating the various

eye movements required in reading mathematics. In conclusion, if teachers would utilize the techniques outlined above, or similar techniques, to provide insight into the language of mathematics and approaches to reading it, their students' success in understanding the concepts of mathematics and in applying those concepts to computations and problem solving should increase.

C. Annotations

Aiken, Lewis R., Jr. Language Factors In Learning Mathematics. Columbus, Ohio: ERIC Document Reproduction Service, ED 068 340, 1972.

Aiken discussed the relationship between language factors in mathematics and mathematics achievement and reviewed relevant research from 1930 to 1972. He examined mathematical, verbal and intellectual abilities; mathematical vocabulary, syntax and readability; techniques and teaching procedures; and problem solving abilities. Aiken contended that mathematics functions as a unique language within itself. He found research underscores the relationship between reading ability and problem solving in mathematics. One teaching strategy Aiken discussed was the verbalization of mathematics problems both by the teacher and the student. An extensive bibliography was included.

Collier, Calhoun C. and Redmond, Lois C. "Are You Teaching Kids To Read Mathematics?" The Reading Teacher Vol. 27 No. 8 (May, 1974): 804-809.

The authors felt success of students in mathematics is directly related to their ability to read and interpret written material. They contended "poor" readers are not the only ones who have problems with the language of mathematics, but "good" readers, as well, often experience reading difficulties in mathematics. The researchers stated different skills are needed in mathematics than in other subjects. They discussed reading factors such as the abstract and concise language of mathematics, style of reading required in mathematics-including the reading rate and eye movements necessary, vocabulary development, and symbols. Several examples of activities for the mathematics classroom were included.

Gupta, I.D. "Mathematics is a Language: A Rationale For Mathematics Instruction," International Journal of Mathematical Education In Science and Technology Vol. 11 (January-March, 1980): 29-32.

Gupta explored the implications of mathematics as a language. First, he examined the semiology of mathematics-how signs are created and transformed for meaning. He noted the similarities between the structures of language and mathematics. Second, he discussed the three basic functions of mathematics education. Finally, he examined the implications of mathematics as a language for teaching strategies.

He identified four categories of mathematical symbols: 1) contentive symbols which include numerals or "nouns" such as 0, 1, 2 and pro-numerals or "pro-nouns" such as a, b, d; 2) operative symbols (prepositions) comprised of symbols such as +, -, $\cap$, $\sqrt{}$, log; 3) relation symbols (verbs) such as =, $<$, $>$, $\exists$ and 4) auxiliary symbols like : (such that) and $\forall$ (for all). He concluded effective mathematics instruction should aim at the internalization of symbols and the generation of expressions from that.

Hollander, Sheila K., Reading The Special Language of Mathematics, ERIC Document Reproduction Service, ED 137 726, 1977.

Approximately 85 percent of the learning which takes place in the secondary school, according to the author, is based on reading. Therefore, it is imperative that mathematics students be able to read and understand their textbooks. Hollander pointed out that content area teachers should not assume a transfer of reading skills from the traditional narrative text to the mathematics text. Mathematics teachers can teach their students necessary skills to understand information in the mathematics textbooks in several ways. First, students should be shown how to modify their reading style to understand mathematics materials. Second, special mathematical symbols and special mathematical vocabulary should be taught specifically using as many concrete examples as possible. Specific teaching

techniques include 1) getting the students to discuss the verbal problem they are to work, 2) making certain the teacher does not do all the talking, 3) preparing short answer, multiple-choice pre-tests before teaching a particular concept and 4) being sure students understand technical vocabulary, symbols and abbreviations.

Krulik, Stephen, "To Read Or Not To Read, That Is The Question!" The Mathematics Teacher Vol. 73 (April, 1980): 248-252.

Krulik stated there is a high correlation between reading comprehension and problem solving in mathematics and yet many mathematics teachers overlook the important area of reading in mathematics. He suggested several activities that can be used in the mathematics classroom to help students read mathematics. Activities included deal with 1) directions in which mathematical expressions are read, 2) style of reading mathematics-slowly and carefully, 3) words which have meanings in mathematics differing from everyday conversation, and 4) reading aspects of students' problem solving skills. Krulik advocated using the Cloze procedure to help the students improve comprehension of their mathematics textbook.

Palmer, William S. Teaching Reading In The Content Areas: Three Pitfalls, Three Possibilities. ERIC Document Reproduction Service, ED 088 022, 1974.

Three pitfalls of reading in the content areas include:

1) The reading specialist is the one to deal with teaching reading in the content areas, 2) The best place to begin reading instruction is with extensive training in phonics and 3) Reading is to be taught as little more than just a bag of devices. Three possibilities for improving the teaching of reading should include: 1) Put the cliché "teaching reading as a process" into practice, 2) Build instruction upon significant leads from linguistics and 3) Develop ways to bridge the gap between the student's spoken language, experiences and variety of patterns within printed language. The mathematics teacher may find this article useful because Palmer illuminates pitfalls to be avoided and also suggests possibilities to test out which are directly applicable to the mathematics classroom.

Phillips, Jo. The Problem With Math May Be "Reading". Instructor Vol. 89 (November, 1979): 67-69.

Phillips stated problems with math may actually be difficulties with the language. She contended that word problems and technical vocabulary, a primary target of mathematics reading research, is only the top layer of reading problems in mathematics. Phillips discussed eight interrelated items important for understanding mathematics. They include: 1) Referents for number symbols and other math tokens, 2) Structure of symbols and words for numbers greater than ten, 3) Meaning of

"little" words, 4) Different interpretations for common words, 5) Eye and hand movements in reading mathematical content, 6) Importance of finding out what message students are getting, 7) Importance of basic number facts, and 8) Importance of estimation.

Schell, Vicki J. "Learning Partners: Reading and Mathematics," The Reading Teacher Vol. 35 (February, 1982): 544-548.

The mathematics teacher, according to Schell, must accept the major responsibility for teaching students to read mathematics because development of mathematical skills is closely associated with effective reading in mathematics. Schell stated mathematics is the most difficult subject to read because it contains more concepts per word, sentence and paragraph than any other subject. Included in her discussion of factors which make mathematics difficult to read were: 1) Nature of the language, 2) Specialized vocabulary, 3) Reading mathematical symbols: directionality, graphic materials and 4) Reading mathematical sentences: reading and solving word problems. Numerous strategies and examples were presented.

Sullivan, Kathryn. The Comprehension of Mathematical Language, A Communication Approach. ERIC Document Reproduction Service, ED 197 287, 1980.

Sullivan applied the Dinnan-Lodge matter-relation reading instruction framework to teach students to determine the message intended by symbolic sentences.

In a matter-relation framework the focus is upon the communication represented by the printed message. Words and symbols used in mathematical sentences are classified into matter or relation references. Mathematical symbolic sentences were defined as two or more names for numbers of sets joined together by a relation symbol. For example, in the mathematical sentence $3 \times 4 = 12$, 3, 4, and 12 are classified as matter; and $\times$ and $=$ are classified as relations. Sullivan contended using the matter-relation classification to teach students the meaning of mathematical symbolic sentences has the potential to "make math a meaningful communication rather than a series of symbols committed to memory."

Wadleigh, Merritt Edward. A History of Reading Programs Involving Instruction In The Content Areas, 1966-1981. ERIC Document Reproduction Service, ED 225 145, 1982.

Wadleigh made a broad study of the teaching of reading in the content area. The idea of teaching reading in the content areas can be traced back to the 1930's. Until the 1960's most of the reading instruction in the secondary school was mainly remedial. There is now, however, a move to incorporate reading skills into all content areas. Besides reviewing the history of reading in content areas, some program types were categorized and summarized, criteria were set to gauge the success of the programs, and finally recommendations

were made for the future. This article provided an excellent background for the mathematics teacher in reading in the content areas.

FOOTNOTES

¹Schell, "Learning Partners: Reading and Mathematics,"
p. 544.

²Anthony V. Manzo, The Math Student/The Math Teacher/
The Math Problem (ERIC Document Reproduction Service, ED 114 767,
1975), p. 1.

³Shephard, "Applying Reading Skills To Mathematics," p. 305.

⁴Harold H. Lerch, "Improving Reading In the Language of
Mathematics-Grades 7-12," Improving Reading In Secondary Schools:
Selected Readings, ed. Lawrence Hafner (New York: The MacMillan
Company, 1974), p. 325.

⁵Texas Education Agency, Learning Partners: Reading
and Mathematics, p. 1.

⁶Schell, "Learning Partners: Reading and Mathematics,"
p. 547.

⁷Hafner, "Reading Mathematics: Strategies For Improving
The Reading of Mathematics," p. 257.

⁸Krulik, "To Read Or Not To Read, That Is The Question!",
p. 249.

⁹Aukerman, "Reading In The Mathematics Classroom," p. 198.

¹⁰Schell, "Learning Partners: Reading and Mathematics,"
p. 547.

¹¹Krulik, "To Read Or Not To Read, That Is The Question!",
p. 248.

¹²Georgia State Department of Education, Reading Mathematics
(ERIC Document Reproduction Service, ED 105 407, 1975), p. 20.

CHAPTER III

MATHEMATICAL VOCABULARY AND SYMBOLS

A. Discussion of the Reading Problem

Many researchers have recognized the importance of teaching vocabulary to students in the context of particular content areas. For example, Taschow has noted that Gray and Holmes found that even small amounts of planned teaching of terms and concepts produces "dramatic" improvements in comprehension and content knowledge.¹ Aiken has concluded that knowledge of vocabulary is important in solving mathematical problems.² Investigators such as Pachtman and Riley, Ciani, Earp and Tanner, Phillips, and Sullivan have studied the vocabulary development in mathematics and feel it is an important part of mathematics instruction.³ Kulm has indicated that a student's inadequate vocabulary is partially responsible for reading difficulties in mathematics, but he believes that a student's inexperience with the difficult symbolism of mathematics, rather than a lack of reading ability, is a larger factor.⁴

Shephard has named four types of vocabulary words which can cause difficulties for mathematics students.⁵ The first type is "the technical word unique to a particular area of mathematics." For example, the words sine and cosine are peculiar to trigonometry; linear equations and polynomials are peculiar to

algebra; and isosceles, arc and polyhedron are peculiar to geometry. The second type is "the word which may have a different meaning in mathematical usage than in everyday usage." Included are words such as prime, natural, radical and exponent. The third type is "the word which signals a mathematical process." Exemplars are words such as times, subtract, multiply and difference. The fourth type is "the nontechnical word which is considered to be general and simple." This type often signals a relationship, and includes words such as than, above, about, each, increase, next, step, require, same and whole.

To summarize, understanding the meanings of words and symbols clearly is necessary in learning to read any type of material. If a student cannot understand the vocabulary and symbols of mathematics, not only will he probably not be able to work a given problem, but he probably will not be able to understand the underlying concepts necessary to solve other problems. Thus, although understanding vocabulary and symbols does not guarantee success in mathematics, it is a starting point to overcome reading difficulties in mathematics.

B. Strategies, Activities and Exercises for Overcoming Mathematics Reading Difficulties

The secondary mathematics teacher may use various strategies, activities, and exercises to help students develop an understanding of mathematical words, symbols and concepts. Because vocabulary development and thus concept development is a process, however, Smith and Kepner suggest that the utility of teaching vocabulary is enhanced and has longer lasting effects when it is done in small but regular increments.⁶

One teaching strategy is to have the students read mathematical material and then list terms they do not understand.⁷ Then the entire class could discuss, and define the terms and keep a glossary of these new terms.

Another strategy for understanding concepts and vocabulary would be to give a short answer or multiple choice pre-test to students.⁸ To adapt this idea to an Algebra class, the teacher might include questions such as those listed below.

1. Select the definition of a term

A prime number is

- a) a natural number whose only factors are itself and 1
- b) a whole number greater than one whose only factor is itself
- c) a natural number greater than one whose only factor is itself
- d) a whole number greater than one whose only factors are itself and one

2. Select an example of a term

In which is a prime number shown?

- a) $40 + 8$ c) 29
- b) 52 d) $49 \cdot 4$

3. Select the meaning of a symbol

What does $<$ mean?

- a) parallel to c) equal to
- b) greater than d) less than

4. Select the meaning of an abbreviation

What does GCF mean?

- a) good commutative factor

- b) greatest complement fraction
 - c) greatest common factor
 - d) greatest common fraction
5. Write a number phrase from a combined language form
- a) a number increased by 43 equals 97
 - b) four times a number equals 72
6. Write a combined language form from a number phrase
- a) $4(x+8)$
 - b) $\frac{1}{2}x^2 - 3$

A third strategy is to present new words to the mathematics class through non-mathematical terms and experiences with which the students are familiar. For example, when introducing the slope-intercept form of an equation in an Algebra class, the teacher might point out to the students that when one student bumps into another student in the hall, the first student intercepts the second student. When introducing the word variable, the teacher might have the class discuss the meaning of variable winds.

A fourth exercise is to have students rewrite mathematical statements in textbooks in order to expand the contextual meaning from which the students may develop comprehension in mathematics. For example, an algebra text might define a function as follows: "If to each value of a variable, there corresponds a unique value of a second variable, the correspondence between these two variables is called a function." A student might rewrite that definition in his own words by saying "a function is a relation in which no two ordered pairs have the same first element."

A fifth strategy employs a linguistic approach to vocabulary development in which structural analysis is used. In this process the student identifies familiar or meaningful parts-- prefixes, suffixes, and roots-- of unfamiliar terms. This approach is particularly useful in mathematics because many mathematical terms contain structural elements that give clues to their meaning. For example, words such as equilateral, quadrilateral, parallelogram, triangle, monomial, polynomial and geometry lend themselves to word analysis. One useful activity in structural analysis would be to have the students keep vocabulary cards that define the word, identify its elements and list other words that contain the same elements.⁹ A sample vocabulary card appears below.

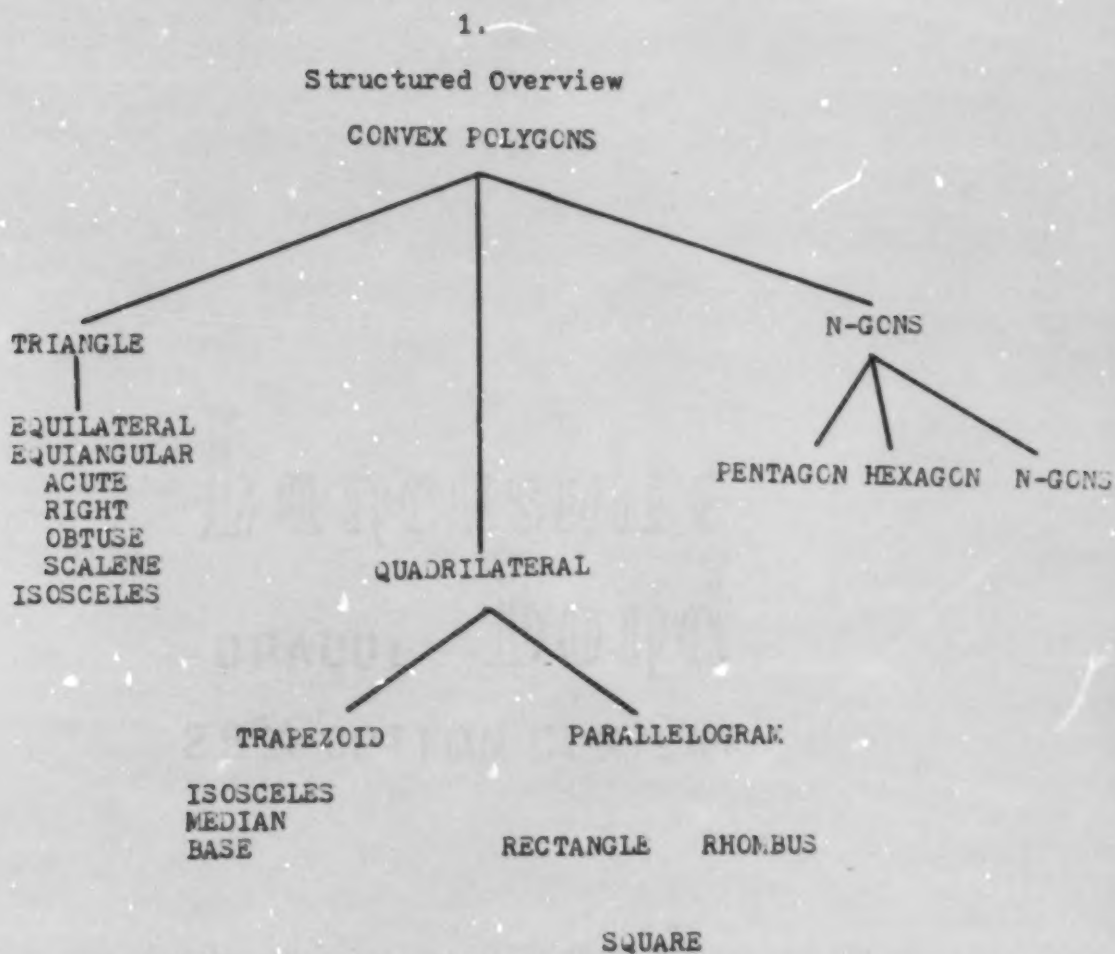
quadrilateral: Quadri/lateral = a polygon with four sides
 Quadri = element meaning "four"
 quadrant, quadrinomial, quadrangle
 Lateral = element pertaining to the side or sides
 equilateral, trilateral, unilateral

A second possible activity in structural analysis would be to examine the vocabulary that is essential to understand a given topic and have the students determine or have the teacher point out the word parts of the vocabulary. One list might include some of the following.¹⁰

ante-	before	$\frac{1}{2}$	hemi, demi
anti-	against	1	uni, mono
auto-	self	2	bi, di
bi-	two	3	tri
circum-	around	4	quadra, tetra
inter-	between	5	penta, quin
mis-	wrong	6	hex, hexa
non-	not		

A sixth strategy to enhance the development of vocabulary and symbols is the structured overview.¹¹ This method is particularly useful for abstract material because it identifies the most important terms in a teaching unit and depicts the relationship between them. By following the structured overview a student can relate new information to what was previously learned. Activity 1 presents an example of a structured overview for use in a geometry class.

Research in vocabulary development suggests that there is no one best way to ensure the understanding of terms and symbols. The following activities, in addition to the strategies previously discussed, are intended to provide a variety of techniques for the secondary mathematics teacher to use to emphasize essential terms and symbols. In addition to helping students to recognize mathematical words and symbols, these exercises and strategies can provide the students with practice in analyzing and explaining the relationships that exist between the words and symbols.



2.

PUZZLE

"What's My Line?" --Properties

Directions: To solve the following puzzle, look at the statements below. Think of a word which fits each description and has the same number of letters as the number of spaces provided in the corresponding line. Write the words on the line. Do this for each statement.

1.			P						
2.			R						
3.			O						
4.			P						
5.			E						
6.			R						
7.			T						
8.			I						
9.			E						
10.			S						

1. Two numbers located the same distance from zero
2. $a(b+c) = (ab) + (ac)$
3. $(a+b) + c = a + (b+c)$
4. The symbol $-a$
5. $a+b = b+a$; $ab = ba$
6. $\sqrt{7}$
7. $a \cdot 1 = a$; $a + 0 = a$
8. Number expressed in the form $\frac{a}{b}$
9. The set of all whole numbers and their opposites.
10. A number added to its opposite equals zero.

3.

Symbol Perception and Definition

Directions: These eighteen cards can be divided into six groups of three each, with each group containing one mathematical symbol, its correct pronunciation, and an example of two values which illustrate the symbol's meaning. Work with a partner and try your luck. When you finish, check with your teacher or another student. Keep a record of the time it takes you to do this.

$<$	equals	twelve / twenty
$=$	is greater than or equal to	5¢ / 1¢
$>$	is less than	10 / \$10.00
$\neq$	is greater than	six / ten
$\leq$	is not equal to	four / 2×2
$\geq$	is less than or equal to	$\frac{10}{2}$ / $\frac{50}{10}$

4.

Algebra Symbol Translation

Directions: Match the algebraic statement on the left with the translation on the right. Collaborate with your work group. Everyone must agree.

___ 1. ab

___ 2. x^2

___ 3. $x^2 - y^2$

___ 4. $(f + s)^2 = f^2 + 2fs + s^2$

a. the square of a number

b. the square of the sum of two numbers equals the square of the first number, plus twice the product of the numbers, plus the square of the second number.

c. the product of two numbers

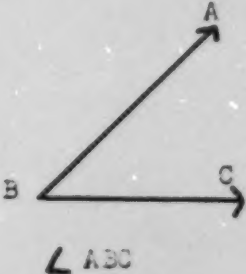
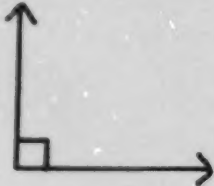
d. the difference of the squares of two numbers

Earle, p. 13

5.

Word/Symbol Relationships

Directions: In each box below there is a mathematical word, term or symbol. When a word or term is present, make a drawing and label it. When a symbol is present, write the expression or make a drawing suggested by it. The first one is done for you.

ACUTE ANGLE 		 ABCD
SCALENE TRIANGLE	ALTITUDE	

Smith and Kepner, p. 32

6.

Equivalent Expressions

Directions: Underline the words or symbols to the right that mean the same as the key word or symbol.

- | | Key word
or symbol | | | | |
|----|-----------------------|------------------|---------------|----------------|------------------|
| 1. | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{3}{6}$ | $\frac{5}{12}$ | $\frac{2}{3}$ |
| 2. | $>$ | greater than | less than | equal to | not equal to |
| 3. | 0.25 | $\frac{25}{100}$ | $\frac{2}{5}$ | $\frac{1}{4}$ | 2 $\frac{5}{10}$ |
| 4. | less than | - | $>$ | $<$ | $\neq$ |
| 5. | $\frac{3x + 9}{3}$ | $x + 9$ | $3x + 9$ | $x + 3$ | $x + 6$ |
| 6. | $\perp$ | perpendicular | parallel | equal | similar |

Smith and Kepner, p. 26

7.

Interpreting Words with Special Meanings

Directions: Write a sentence using each of these words in a mathematical context.

- | | |
|-----------|-------------|
| 1. angle | 6. square |
| 2. degree | 7. set |
| 3. plane | 8. volume |
| 4. odd | 9. solution |
| 5. base | 10. power |

Texas Education Agency, p. 8

8.

Word and Symbol Association

Develop a set of cards with phrases of mathematical symbols. Have a student come to the front of the class and give him or her one of the cards. Do not allow the rest of the class to see what is on the card. The student then reads aloud what is on the card. The other students write the symbols for what they hear. Students can compare their answers.

$$3 + 2$$

$$\frac{1}{7}$$

$$\frac{2}{3} + 4$$

$$\frac{2 + 1}{7} + (1 \times 4)$$

$$\frac{(X + Y)^2}{(X + Y)}$$

$$\text{LOG}_3 X = 2$$

9.

Dual Meanings for Symbols

Have the students discuss the meaning of "-" in each of the following:

1. $\overline{AB}$
2. $8 - 2$
3. $- 4$
4. $3.\overline{3}$

Florida State Dep. of Ed., p.9

10.

Directionality of Symbols

Make a wall chart that contains some symbols that may create a problem in reading. Guide the students through a discussion of the directions they take in reading them. With a colored pen, draw arrows to indicate the directions taken. In addition to drawing arrows, write out, in English, the expression. Discuss this chart and leave it up for further reference.

Examples:

- | | |
|--|--|
| 1. 5^2 | "the square of five" |
| 2. 5^2 | "five squared" |
| 3. $3x$ | "three times x" |
| 4. $5 \div 2$ | "five is to two" |
| 5. $4 \overline{)12}$ | "twelve divided by four" |
| 6. $\frac{1}{2} + 1$ | "one-half plus one" |
| 7. $\begin{array}{r} 6 \\ -2\frac{1}{3} \\ \hline \end{array}$ | "six minus two and one third is equal to what number?" |
| 8. $7 - \boxed{} = 3$ | "what number subtracted from seven leaves three?" |

Florida State Dep. of Ed., p. 10

11.

Definitions

Give some definitions, real or contrived, and have students use them to answer pertinent questions about main points in the definition.

TRAPEZOID: A quadrilateral that has only one pair of opposite sides parallel.

- a. How many sides does it have? How do you know?
- b. Can any quadrilateral be a trapezoid? Why or why not?
- c. Does this definition differ from the one in your textbook? How?

Florida State Dep. of Ed., p. 13

"Punctuation Marks" of Mathematics

Make a poster of the following punctuation marks of mathematics and have the class provide examples in which the marks might be used in mathematics.

$+$	$-$	$\div$	$\cdot$	$=$	$\neq$	$\times$	$<$	$>$
$\gcd$	$\cos$	$\tan$	$\sin$	$\rightarrow$	$\leftarrow$	$\pm$	$\downarrow$	$\checkmark$
$^{\circ}$	$\textcircled{C}$	$\textcircled{R}$	Δ	$\square$	$\neq$	$\cong$	$[]$	$()$
$\dagger$	$\mp$	$\{\}$	π	∞	μ	$\therefore$	$\mathcal{B}$	Δ

12.

Aukerman, p. 199

41

37

13.

FIFTY WORDS AND PHRASES ASSOCIATED WITH BASIC PROGRAMMING ARE HIDDEN IN THIS ARRAY. THEY ARE SPELLED HORIZONTALLY, VERTICALLY, DIAGONALLY, BOTH FORWARD AND BACKWARD. HOW MANY CAN YOU FIND?

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	N	O	I	T	C	N	U	F	C	O	U	N	T	E	R
2	A	D	L	O	V	L	I	N	A	T	L	B	S	G	
3	P	O	R	O	A	A	R	D	B	I	G	E	O	D	B
4	C	L	O	S	L	O	M	I	N	R	I	T	N	A	C
5	L	O	R	S	U	M	R	R	P	B	C	S	A	B	A
6	E	R	E	T	E	E	E	A	O	H	A	E	I	I	H
7	S	W	P	R	I	F	A	R	Y	T	M	S	E	C	T
8	T	U	E	I	N	I	R	I	N	O	S	E	D	O	R
9	H	A	P	l	G	D	X	R	A	R	E	N	O	C	S
10	A	I	N	E	O	E	Y	T	P	E	N	I	C	H	R
11	N	M	R	M	U	C	T	R	L	E	T	S	L	N	E
12	F	O	A	H	X	G	R	O	F	E	R	M	O	E	L
13	T	R	S		U										
14															
15															
16															

C. Annotations

Carpenter, Thomas P., Corbitt, Mary Kay; Kepner, Henry S., Jr., Lindquist, Mary M. and Reys, Robert E. "Student Performance in Algebra: Results From The National Assessment." School Science and Mathematics Vol. 82 No. 6 (October, 1982): 514-531.

The authors reported student performance in several phases of algebra: symbolization, algebraic representation or translation and algebraic manipulations. Several problems were noted. Student performance on exercises requiring translation to an algebraic format was not high. For all the time given to solving linear equations in algebra, overall student performance was weak. Results in manipulating formulas and problem solving were poor. The authors concluded students have difficulty manipulating mathematical symbols in expressions and equations. They contended, however, this study of student performance on algebra topics does not suggest a complete failure. A cluster of 15-20 percent of the 17-year-old population showed mastery of algebraic skills and concepts as evidenced by the average performance levels on the most difficult exercises. In addition, about 30-40 percent of the population showed an intuitive knowledge of algebraic processes.

Ciani, Alfred J., "Mastering Word and Symbol Language In Mathematics," School Science and Mathematics Vol. 81 (May/June, 1981): 371-377.

Ciani gave several educators' views that mathematics language is comprised of mathematical symbols and word symbols. The ability to use both is necessary for success in mathematics. Ciani referred to the reading process as a hierarchy of skills: word recognition, literal understanding, interpretive meaning, and application of meaning. It was stressed that recognition and pronunciation must take place before understanding or learning is possible. Ciani suggested activities as a method of strengthening learning in the areas of sight word, context clues, structural analysis, phonic clues, and dictionary skills.

Dunlap, William P., and McKnight, Marth Brown. "Vocabulary Translations for Conceptualizing Math Word Problems," The Reading Teacher Vol. 32 (November, 1978): 183-189.

Dunlap and McKnight believed that if math teachers are going to help children with word problems the teachers must understand the reading and thinking process involved. They discussed two important problems affecting children's ability to solve math word problems. The first problem is that a three level translation of vocabulary is required in understanding math word problems. The three levels are the general, technical, and symbolic vocabularies. Second, the child has trouble conceptualizing the problem. In order to understand the message of the word problem, the children must understand the parts of each vocabulary,

be able to translate from one vocabulary to another and be able to think in each vocabulary. These translation processes and thinking processes within each vocabulary are critical to conceptualizing the word problem in mathematics.

Earp, N. Wesley, and Tanner, Fred W. "Mathematics and Language," Arithmetic Teacher Vol. 28 (December, 1980): 32-34.

The authors focused on three elements related to reading in mathematics: 1) language and mathematics, 2) readability and mathematical materials, 3) suggestions for improving reading in mathematics. Several conclusions and recommendations were presented. First, children's language pattern may contribute to or detract from learning mathematics. Second, the language of non-mathematical terms is much more familiar to children than the language of mathematical terms. Third, more oral language should be used in the mathematical classroom. This feat might be accomplished by introducing new words and symbols by a hear-see-say in syllables and say in sentences method. Fourth, a Cloze procedure might be used to give the teacher a general idea of the students' general comprehension level of text. A fifth recommendation was to use a language-experience approach for children to work word problems. In this approach children rewrite problems in their own words.

Florida State Department of Education, "Vocabulary and Symbols,"
Reading the Language of Mathematics ERIC Document
 Reproduction Service, ED 134 972, 1975 1-18.

The authors contended that vocabulary and symbols in mathematics are abstractions and that before presenting them to students teachers need to present to students ideas with concrete experiences. They suggested a laboratory approach for the student to experience terms and phrases before paperwork. Three competencies for reading mathematical symbols were suggested:

1) students need to be able to associate a word or words with a symbol, 2) students should be able to express an idea in pictures, words, objects and symbols, and 3) they need to note there may be more than one meaning for some symbols. Directionality was also considered in reading mathematical symbols. Reading words in mathematics involves developing word meanings by using prefixes, suffixes, root words, context clues and definitions. Another consideration in reading mathematical words is interpreting words with special meanings. The authors included excellent activities and suggestions for the mathematics classroom.

Milligan, Constance F. and Milligan, Jerry L. "A Linguistic Approach To Learning Mathematics Vocabulary," The Mathematics Teacher Vol. 76 (October, 1983): 488-490.

The authors believed vocabulary development is an integral part of mathematics instruction. They recommended a linguistic approach, sometimes referred

to as structural or word analysis, to learning the vocabulary of mathematics. Two advantages of the linguistic approach to vocabulary development over methods that do not focus on word elements were presented. First, the authors' method enables the student to discover word study can be exciting. Second, an awareness of suffixes, prefixes and roots is developed. Thus, these word parts can be used to determine the meanings of other unfamiliar words. The authors recommended that vocabulary development in mathematics be continuous.

Pachtman, Andrew B., and Riley, James D. "Teaching the Vocabulary of Mathematics Through Interaction, Exposure and Structure," Journal of Reading Vol. 22 (December, 1978): 240-244.

Understanding the vocabulary in mathematical word problems is a big hindrance for students. Typically, teachers' "talk and chalk" a word problem with the teachers telling the students the meanings of terms in the problems. The authors contended, however, that students need some kind of activity to expose and reinforce them to the terminology in order for them to internalize the terms. The structured overview is the means by which the students can see the overall picture and interrelationships of necessary terms and concepts. The authors presented nine steps for constructing a structured overview to teach vocabulary related to specific word problems.

Smith, Cyrus P., Jr., and Kepner, Henry S., Jr., "The Development of Mathematics Vocabulary," Reading in the Mathematics Classroom ERIC Document Reproduction Service, ED 207 836, 1981 .

Smith and Kepner cited six strategies that a reader uses to unlock unfamiliar words and symbols for meaning and pronunciation. These are: phonetic analysis (sound to sound pronunciation), instant recognition (automatic response), syllabic analysis (speech sounds associated to vowels), structural analysis (utilizes roots, prefixes, suffixes, compounds), contextual analysis (anticipation of meaning by association with other words), and dictionary or glossary usage (reference skills). Instruction in vocabulary can complement any curriculum when placed within the proper perspective. Vocabulary teaching has greater utility and longer lasting effects when it is done in small but regular increments (five minutes daily is recommended). "List-Group-Label" is a strategy Smith explained which helps students organize concepts and analyze relationships. Smith gave examples of activities to reinforce learning skills in vocabulary development in mathematics.

Taschow, Horst G. Reading In Mathematics ERIC Document Reproduction Service, ED 046 670, 1970

Taschow focused on difficulties inherent in the reading of mathematics at the secondary and college levels. Emphasis was placed on: 1) understanding verbal and

mathematical symbols, 2) understanding arithmetic numerals, literal numbers, operational symbols and expressions of relationships. The author included specific examples from algebra and arithmetic in his discussion. Taschow advocated teaching technical vocabularies and their special concepts and various mathematical symbols in a direct, systematic and planned fashion. He recommended using Group Informal Reading Inventory, Directed Reading Activity and a multi-sensory approach for improving reading efficiency in mathematics.

Texas Education Agency, "Specialized Vocabulary and Symbols,"
Learning Partners: Reading and Mathematics ERIC
Document Reproduction Service, ED 179 422, 1979 .

This pamphlet appeared to be based on the same outline as the Florida State Department of Education, 1975. The authors pointed out the importance of students not merely memorizing and vocalizing word meanings. They must develop ideas and concepts represented by the word. The authors felt students need to use context clues if they are to read mathematics with meaning. Symbols were defined as a character, abbreviation or mark which represents an expression, idea, concept, quantity or operation. Activities for developing and assessing students' understanding of mathematical words, symbols and concepts were presented.

FOOTNOTES

¹Taschow, Reading in Mathematics, p. 5.

²Aiken, Language Factors In Learning Mathematics, p. 1.

³Alfred J. Ciani, "Mastering Word and Symbol Language In Mathematics," School Science And Mathematics Vol. 81 (May/June, 1981); pp. 371-377; N. Wesley Earp and Fred W. Tanner, "Mathematics and Language," Arithmetic Teacher Vol. 28 (December, 1980); pp. 32-34; Andrew B. Pachtman and James D. Riley, "Teaching the Vocabulary of Mathematics Through Interaction, Exposure and Structure," Journal of Reading Vol. 22 (December, 1978); p. 240-244; Jo Phillips, "The Problem With Math May Be Reading," Instructor Vol. 89 (November, 1979); pp. 67-69; Kathryn Sullivan, The Comprehension of Mathematical Language, A Communication Approach (ERIC Document Reproduction Service, ED 197 287, 1980); p. 10.

⁴Gerald Kulm, "Sources of Reading Difficulty In Elementary Algebra Textbooks," The Mathematics Teacher Vol. 66 (November, 1973), p. 649-652.

⁵Shephard, "Applying Reading Skills To Mathematics," p. 307.

⁶Smith and Kepner, Reading In the Mathematics Classroom, p. 24.

⁷Hollander, Reading The Special Language of Mathematics, p. 12.

⁸Ibid., p. 13.

⁹Constance F. Milligan and Jerry L. Milligan, "A Linguistic Approach To Learning Mathematics Vocabulary," The Mathematics Teacher Vol. 76 (October, 1983), p. 489.

¹⁰Richard A. Earle, Teaching Reading and Mathematics (Newark, Delaware: International Reading Association, 1976), p. 325.

¹¹Smith and Kepner, "The Development of Mathematics Vocabulary," p. 15.

CHAPTER IV

WORD PROBLEMS

A. Discussion of the Reading Problem

Many secondary level mathematics teachers have encountered the groans of students when word problems are mentioned in mathematics. Students often find the solution of word problems to be the "most complex and troublesome part of mathematics."¹ There are many reasons for this difficulty. Among these are the fact that several competencies are needed to read and solve word problems.² First, the students need to be prepared to read word problems. The student must be able to read and understand the word problem adequately. Readiness for mathematics reading requires that the student can perform the algebraic and arithmetic operations necessary to solve the word problems. Second, the students need to be able to analyze the information in word problems. The student should be able to state relationships and needed operations and determine what is being asked. Third, the students need to be able to translate and use signal words which indicate particular operations, such as less, increase, and twice. They must be able to translate the printed words into mathematical sentences. Fourth, the students need to be able to identify sufficient, insufficient, and extraneous information in a word problem because sometimes, for example, there is not

enough information to solve a problem and other times there is excess information. Fifth, the students should be able to employ some systematic strategy that will help them attack and solve the word problem.

In addition to the competencies needed to solve word problems, word problems present other special reading difficulties to the mathematics student. Researchers have identified many of these. Smith and Kepner maintain that reading difficulties stem from the students' inadequate experience with specific mathematics vocabulary used in word problems.³ Call and Wiggin indicate that teaching how to analyze and solve word problems involves implementing reading study skills rather than mathematical skills.⁴ Riley and Pachtman feel students need specific guidance to synthesize and translate information, sift out important information and perceive often hidden relationships.⁵ Similarly, Earle contends that students lack the ability to synthesize and transfer general directions to problem solving.⁶ According to Earle, students must "perform accurately all the previous levels of reading--perceiving symbols, attaching literal meaning and analyzing relationships-- and extend that knowledge toward the correct or creative solution of a mathematical problem, actual or simulated."⁷ Aukerman suggests four factors which account for students' inability to read and interpret verbal problems.⁸ These factors include an inadequate knowledge of mathematical shorthand and punctuation, low reading ability, inadequate knowledge of mathematical processes, and the fact that the problems themselves may be poorly written. According to these

researchers' findings, instruction in mathematics reading should help to improve a student's ability to solve word problems.

In contrast, however, Knifong and Holtran do not believe that working on improving reading skills will help to correct word problem difficulties.⁹ They found that student failures in solving word problems are due to clerical or computational errors rather than reading problems.

Thus, as most researchers have indicated, solving word problems is a very difficult task, which is even more difficult for a student with a mathematics reading deficiency. Teachers therefore must recognize this problem and strive to help their students become more proficient in the reading skills needed to analyze and solve word problems.

B. Strategies, Activities and Exercises for Overcoming Mathematics Reading Difficulties

A variety of strategies, activities and exercises may help students overcome the difficulties presented by word problems. One strategy for overcoming difficulties in reading problems is to have the students group words in the problems that relate to the same process. For example, the words - increase, plus, sum and longer than - relate to addition. A similar exercise is to have the students identify the key words in a verbal phrase and then identify the operation indicated by those words. The student might construct a table such as the one shown in Activity 1. A third aid is to have the students translate written word phrases into mathematical sentences and also to have the students translate

verbal phrases into algebraic expressions (See Activity 2). This will give the students needed practice in translations that can be applied to word problems. Shephard suggests another activity in which students work problems without numbers.¹⁰ For example, a teacher might ask the students, "What unit of measure would you use to measure the area of page 98 in your book? The area of New York City? The area of your classroom floor?" A fifth activity would be to have students write a word problem on paper and indicate the extraneous information by underlining, crossing out or circling the extra words not considered essential to understanding the problem. A sixth aid for a student who has difficulty analyzing and synthesizing verbal problems is to provide the student with some strategy that he may employ to solve the word problem. Robinson suggests that teachers use strategies which emphasize how to read and study problems rather than how to solve problems.¹¹ Several researchers have proposed some strategies. Shephard suggests a directed reading method for solving word problems.¹² His method consists of the following six steps: 1) Read the problem slowly and carefully, 2) Determine what is asked, 3) Reread the entire problem to determine the facts, 4) Determine the mathematical processes to use, 5) Estimate the answer and judge whether it is reasonable, and 6) Compute the answer. (An application of these steps to a word problem is found in Activity 3). Riley and Pachtman believe "telling them what to do is not telling them how to do it."¹³ They suggest using a reading guide with word problems which

helps the students comprehend on a literal, interpretative and applied level.¹⁴ At the literal level the student is concerned with the facts of the problem. At the interpretative level the student comprehends the mathematical concepts or ideas underlying the problem. The numerical depictions related to the problem are involved at the applied level. (See Activity 4 for a sample word problem reading guide.) This method does not assume that the student can understand and apply appropriate concepts as does the Directed Reading Method. The reading guide also helps encourage students to see relationships among the different aspects of the problem. Maffei offers a method for analyzing and applying data in word problems which is a variation of a reading study skill procedure developed by other reading specialists.¹⁵ The method is called PQ4R----Preview, Question, Read, Reflect, Rewrite, and Review. This method stresses careful reading and rereading of the problem with special emphasis on the meanings of mathematical words and phrases and their relationship to the whole problem. (A sample PQ4R outline sheet can be found in Activity 5).

1.

phrase	key word	operation	symbol
eight times a number	times	multiplication	x
a number increased by three	by increased	addition	+
a number decreased by three-fourths	decreased by	subtraction	-
twice the amount	twice	multiply by 2	2 x

2.

Translations

Directions: Write an algebraic expression for each of the following.

1. Three times some number
2. A number decreased by five
3. The product of 7 and the result of subtracting 8 from some number
4. An even number
5. An odd number

Directions: Translate each word sentence into a mathematical sentence.

6. Three times y equal 36
7. The difference of z and 4 is 13
8. The sum of 5 and twice k is 11
9. The sum of twice m and seven is less than 23
10. Eight is one less than the product of 6 and some number
11. Thirty-one is three more than the product of seven and t
12. The quotient of n and 7, decreased by 5, is 1

3.

Directed Reading Method

Problem: Mr. Stone bought a new sewing machine for his family. The store asked him to make a down payment of 20%. If the cost of the machine was \$160.00, how much was the down payment?

1. The situation: Mr. Stone is buying a sewing machine and he must make a specified down payment.
2. What is asked: How much is the down payment? How much is a key term. Does it indicate process?
3. Facts given: The sewing machine cost--- \$160.00
Down payment must be 20%
Underlined terms may be ones on which students will need specific clarification.
4. Process: Multiplication to compute the size of the down payment.
5. Estimate: A little more than \$30.00
(20% of 100 is 20.00; 20% of 50 is 10.00)
6. Compute the problem: 20% of 160

Shephard, p. 316

4.

Word Problem Reading Guide

Problem: A printing press which Ralph works can print 15,000 sheets of paper per hour. How many sheets can the press print per minute?

Part 1. Facts of the problem

Directions: Read the word problem above. Then, under column A, check those statements that contain the important facts of the problem. Refer back to the problem to verify your responses. Under column B, check those statements you think will help you solve the problem.

A	B	
Facts	Will Help	
_____	_____	1. The printing press can print 15,000 sheets per minute.
_____	_____	2. The printing press can print 15,000 sheets per hour.
_____	_____	3. The problem asks how many sheets can be printed in one minute.
_____	_____	4. Ralph works at a printing company.

Part II. Mathematical ideas and interpretation.

Directions: Check the statements that contain math ideas related to this problem. Refer back to Part I (Column B) to verify your responses. (You may wish to change some of your responses in Part I)

_____	1. There are 60 seconds in one minute.
_____	2. The word "per" often suggests division.
_____	3. Division is the opposite of multiplication.
_____	4. "Per" may suggest a proportion.
_____	5. There are 60 minutes in one hour.
_____	6. Since a minute is less than an hour, the answer will be less than 15,000.

Part III. Numerical depictions.

Directions: Below are possible ways of computing a solution. Check those that apply to this problem. Refer back to Parts I (Column B) and II to verify your responses. (You may wish to change some of your responses in Part I or II.)

_____	1. $60 \times ? = 15,000$
_____	2. $\frac{15,000}{60} = ?$
_____	3. $15,000 \div 60 = ?$
_____	4. $15,000 \times 60 = ?$
_____	5. $\frac{15,000}{?} = 60$

Based on your responses to Parts I, II, and III, compute the answer to the problem.

PQ4R Outline Sheet

1. Preview:
 - a. First reading of problem
 - b. List unknown words and phrases for possible discussion:
 - 1) consecutive
 - 2) even
 - 3) integers
 - 4) _____
2. Question:
 - a. Second reading
 - b. Write direct question of problem:
Find 3 consecutive even integers
3. Read:
 - a. Third reading
 - b. List all word facts of problems in some logical order:
 - 1) 4 times the first
 - 2) decreased by the second
 - 3) is 12 more than
 - 4) twice the third
 - 5) _____
4. Reflect:
 - a. Fourth reading
 - b. What is x , the unknown quantity, representing?
Let $x = 1st\ consecutive\ even\ integer: x, x+2, x+4$
 - c. Translate word facts into algebraic facts with the use of x .
 - 1) $4(x)$
 - 2) $-(x+2)$
 - 3) $= 12 +$
 - 4) $2(x+4)$
 - 5) _____
5. Rewrite:
 - a. Rewrite algebraic facts in terms of a "balanced" equation and then solve for x :

$$4x - (x + 2) = 12 + 2(x + 4)$$

$$=$$

$$x = 22$$
6. Review:
 - a. Substitute value of x in equation to check for true sentence: Check: $4(22) - (22 + 2) = 12 + 2(22 + 4)$
 - b. Does the problem make sense in terms of the question?
22, 24, 26 are 3 consecutive even integers such that....

6.

Study Guide Based on a Word Problem

Word Problem: In a sum of money there are three times as many dimes as nickels. If the total amount of money is \$7.00, how many dimes are there?

True or False:

1. (Literal comprehension)

- ☐ a. The problem speaks of dimes and nickels.
- ☐ b. There are three times more dimes than nickels.
- ☐ c. The nickels plus the dimes equal \$7.00. (700 cents)

2. (Interpretive)

- ☐ a. The problem asks the total value of the dimes.
- ☐ b. An unknown number can be represented by X.
- ☐ c. X can represent the number of nickels.
- ☐ d. The problem asks the total number of dimes.
- ☐ e. $X \cdot 5$ could stand for the amount of money in nickels.
- ☐ f. What operations must be used to solve the problem?

Answer:

3. (Applied)

- ☐ a. What is the answer to the problem?
- ☐ b. If there are one fourth as many nickels as quarters and the total was \$4.20, how many quarters are there?
- ☐ c. Make up a similar problem.

Hafner, p. 266

7.

Analyzing Word Problems

The teacher can select a word problem that requires the students to think in sequence. Each sentence should be written on a separate cardboard strip and put in an envelope. The student can then open the envelope, take out the strips and arrange them in order. If the teacher would put the numbers on the back of the strips, the student can check his own work. These strips can be made to use with a flannel board or they can be pinned on a cork board. If all students in a class are having problems in this area, the teacher can give each one an envelope and when he finishes, he can swap his envelope with someone else.

Sample problem:

Carmen worked as a cashier in a department store one summer. One week, she worked long hours. On Monday, Tuesday, and Wednesday, she worked 9 hours per day. On Thursday, she worked 8 hours. On the other two days she worked 5 hours each. At \$2.25 per hour, how much money did she earn that week?

8.

Directions: Circle the following items that could be combined for each operation. Only numbers with common units may be added together.

- | | | | | |
|-----------------|--------------------|---------------------|---------------------|---------|
| 1) addition: | 7 hours | 2 days | 4 gallons | 8 hours |
| 2) subtraction: | 8 days
per year | 9 weeks
per year | 2 days per
year | 3 days |
| 3) division: | 9 miles | 2 miles | 3 hours | 4 |
| 4) division: | 9 miles | 2 miles | 3 miles per
hour | 5 hours |

Note: Answers may vary on 3) and 4)

9 miles: 3 hours = 3 miles per hour

9 miles: 3 miles per hour = 3 hours

Florida State Dept. of Education, p. 30

Have students read verbal problems and supply the information asked for in the following chart:

	What information is needed?	What terms need defining?	What operation is needed?	What is the order of operations?	What variables are given?
Problem 1					
Problem 2					
Problem 3					
Problem 4					

Cloer, Thomas, Jr. Factors Affecting Comprehension of Math Word Problems-A Review of the Research ERIC Document Reproduction Service, ED 209 655, 1981 .

Cloer focused on students' comprehension of math word problems from a pedagogical perspective, a math educator's perspective and a reading educator's perspective. From the perspective of the math educator Cloer suggested that to improve comprehension one must concentrate on the teaching of concepts, procedure, generalizations, logical exchanges, and number facts. From the reading educator's perspective difficulty with word problems would relate to linguistic and psycholinguistic comprehension theories. Vocabulary development and literal interpretation are both important. Also, textbook readability is a major factor to the success of the student. The pedagogical perspective relates to the variables included or deleted in a word problem. Several of these variables are absence of a diagram, presence of unrelated information, and numbers introduced in an order other than needed for the solution of the problem.

Florida State Department of Education, "Reading and Solving Word Problems," Reading the Language of Mathematics ERIC Document Reproduction Service, ED 134 972, 1975 : 27-35.

Reading and solving word problems presents difficulties for many mathematics students. Activities were presented to promote the following competencies in reading and solving math word problems: readiness for reading

word problems, students need to be able to analyze information in word problems, they should be able to translate and use signal words, students should be able to identify insufficient, sufficient, and extraneous information in a word problem and they should employ some strategy that will help them solve word problems.

Knifong, J. Dan and Holtran, Boyd D. "A Search for Reading Difficulties Among Erred Word Problems," Journal for Research in Mathematics Education Vol. 8 No. 3 (May, 1977): 277-230.

The authors believed "the issue (of reading and word problem difficulties) is not whether reading underlies success in word problems-- that is assumed. Rather, the issue is whether poor reading skills typically contribute to failure, or whether failure is due to some other factor(s)." The authors found little evidence of poor reading abilities affecting children's success in solving word problems. Instead, they attributed student failures in mathematical word problems to computational errors. The recommended teachers help students develop communication skills and do not expect work on reading skills to correct word problem difficulties.

Maffei, Anthony "Reading Analysis In Mathematics," Journal of Reading Vol. 16 (April, 1973): 546-549.

Maffei cited the study by Call and Wiggin (1966) as providing relief to both students and teachers in dealing with reading difficulties with math word

problems. This study indicated that teaching of mathematical word problems is implementing reading study skills. The author adapted Thomas and Robinson's study skills device - PQ4R to mathematical word problems to provide a reading study attack that is useful to students. The components are Preview, Question, Read, Reflect, Recite and Rewrite. The emphasis of this procedure is on the meanings of mathematical words and phrases and their relationship to the whole problem.

Manzo, Anthony V., The Math Student/The Math Teacher/The Math Problem ERIC Document Reproduction Service, ED 114 767, 1975 .

Manzo examined the problem of mathematical illiteracy by exploring three zones: the math student, the math teacher and the math problems. Several facts were noted in each of these areas.

The math student.....

- 1) To most students math seems unrelated to requirements of daily life
- 2) Students seldom receive any satisfaction from attempting word problems as they usually fail to solve the problem.
- 3) Many students are impeded by the four "D's": defects (being unable to learn easily and effectively), deficiencies (missing subskills), disruptions (high anxiety, tension), and differences (unusual cognitive style).

The math teacher.....

- 1) Math teachers are usually less verbal and socially responsive than other teachers.
- 2) Math is taught in a specific way in a specified order with little room for thinking and experimenting.

Math problems.....

- 1) Word problems contain new words and notation that confuse and frustrate the reader.
- 2) Word problems often use complex language which loses the reader.
- 3) Word problems are not related to students' life situations.

Riley, James D. and Pachtman, Andrew B. "Reading Mathematical Word Problems: Telling Them What To Do Is Not Telling Them How To Do It." Journal of Reading Vol. 21 (March, 1978): 531-533.

Riley and Pachtman contended that mathematical word problems present new areas of difficulty because the language of word problems is compact and because mathematical concepts and relationships are often "hidden" to the student. The authors encouraged the use of a three level reading guide which enlightens the students' understanding at the literal, interpretative and applied levels. The reading guide consists of the following:

- 1) the facts of the problem (literal level), 2) the mathematical concepts underlying the problem (interpretative level), and 3) the numerical depictions related to the problem (applied level).

Smith, Cyrus F., Jr., and Kepner, Henry S., Jr., "The Dilemma of Word Problems," Reading in the Mathematics Classroom ERIC Document Reproduction Service, ED 207 836, 1981 : 39-41.

Smith cited a major reason for students' poor performance on word problems is that students cannot read. He then cited evidence to disprove this statement and concluded that students can read but have difficulty synthesizing the available information. Smith presented strategies for solving word problems : students break the problem into components, then they are to synthesize the information. Smith offered Earp's five steps in reading word problems. The steps include: read to visualize the overall situation, read to get specific facts, note difficult vocabulary and concepts, reread to plan the solutions, and reread the problem to check the procedure and solution.

Texas Education Agency, "Reading and Solving Word Problems," Learning Partners: Reading and Mathematics ERIC Document Reproduction Service, ED 179 422, 1979 : 23-27.

This pamphlet appears to be based on the same outline as the Florida State Department of Education, 1975. Topics such as preparing to read word problems, analyzing word problems, identifying extraneous information and using problem solving strategies were discussed. Activities designed to help students develop skills in translating word problems are presented by the authors.

FOOTNOTES

- ¹H. Alan Robinson, Teaching Reading and Study Strategies The Content Areas (Boston, Mass.: Allyn and Bacon, Inc., 1975), p. 181.
- ²Florida State Department of Education, "Comprehension," Reading The Language of Mathematics, p. 27.
- ³Smith and Kepner, "The Dilemma of Word Problems," Reading In The Mathematics Classroom, p. 39.
- ⁴Russell J. Call and Neal A. Wiggin, "Reading and Mathematics," The Mathematics Teacher Vol. 59 (February, 1966) p. 149-157.
- ⁵Riley and Pachtman, "Reading Mathematical Word Problems: Telling Them What To Do Is Not Telling Them How To Do It!", p. 531.
- ⁶Earle, Teaching Reading and Mathematics, p. 57.
- ⁷Ibid., p. 49.
- ⁸Aukerman, "Reading In The Mathematics Classroom," p. 206.
- ⁹J. Dan Knifong and Boyd D. Holtran, "A Search for Reading Difficulties Among Erred Word Problems," Journal for Research In Mathematics Education Vol. 8 No. 3 (May, 1977): p. 230.
- ¹⁰Shephard, "Applying Reading Skills To Mathematics," p. 317.
- ¹¹Robinson, Teaching Reading and Study Strategies, p. 181.
- ¹²Shephard, "Applying Reading Skills To Mathematics," p. 316.
- ¹³Riley and Pachtman, "Reading Mathematical Word Problems: Telling Them What To Do Is Not Telling Them How To Do It!", p. 531.
- ¹⁴Ibid., p. 532.
- ¹⁵Anthony Maffei, "Reading Analysis in Mathematics," Journal of Reading Vol. 16 (April, 1973): 547.

CHAPTER V

MATHEMATICAL READING SKILLS

A. Discussion of the Reading Problem

The secondary mathematics student is confronted with unique reading tasks. Because the special language of mathematics is concise and abstract, the student must understand the verbal vocabulary of mathematics and the mathematical symbols. The mathematics student also must analyze and interpret material such as graphs, formulas, statistical reports and word problems. Thus, the mathematics student needs different skills and techniques when reading mathematics than when reading other curriculum materials.¹ Seven reading skills apply to mathematics, some of which already have been discussed. The first four skills are unique to mathematics. The first skill is careful reading of the succinct language of mathematics. The second skill is understanding the mathematical vocabulary and interpreting mathematical symbols and abbreviations. The third skill is the ability to solve word problems by using the necessary analytical and computational steps. The fourth skill is proficiency in the special reading tasks of mathematics, such as equations, charts, graphs and tables. The remaining three skills, although important in mathematics, are not unique to that subject. They are: 1) following directions, 2) relating information already

learned to what is currently being read and 3) selecting reading methods appropriate for the material being read.²

The mathematics teacher has the responsibility of teaching her students how to read mathematical material while at the same time teaching the students the concepts of mathematics and their applications. By teaching the students specialized reading skills and practicing the principles of reading instruction during classroom activities, the teacher can improve the students' chances of success in developing reading skills and increasing comprehension, thus improving their achievement in mathematics.

B. Strategies, Activities and Exercises for Overcoming Mathematics Reading Difficulties

The secondary mathematics teacher can employ numerous reading techniques, strategies for comprehension, and activities in the mathematics classroom. First, the mathematics teacher can aid the student in reading and interpreting materials such as tables, graphs and charts. The Texas Education Agency suggests that when a student is reading graphic material he should first survey the entire graph to determine what it is about, how it is organized, and what information is supplied.³ The mathematics student reading a statistical table can follow three steps suggested by Shephard to aid in understanding the table.⁴ First, he should read and understand the statement that tells what the table represents. Second, he should study each vertical column of items and read their headings to determine what they represent. Third, he should study the vertical columns thoroughly

to compare or draw conclusions. In addition to giving specific directions for reading the table, the teacher should present several applications of the table. For example, if the student is given a table of trigonometric ratios, he must understand the abbreviations "sin", "cos", and "tan"; the order of degrees presented in the table and how to find the tangent of a 34.4 degree angle, for instance. Four steps have been suggested by Shephard to help students read various types of graphs -- bar, circle, and line.⁵ First, the student should read the phrase beneath the graph so he will know what is supposed to be pictured. Second, the student should read the numbers to the left of the graph so he can analyze and note what is being measured. Third, the student should read the letters at the bottom so he can analyze and note the quantity or units of measurement. Fourth, the student should note the name and amount of the fractional part of each section. If the graph is a pictograph, the student will need to be able to use the key to translate each picture into numbers.

Second, the teacher may help her students learn to follow directions by suggesting the following steps: 1) Read the directions carefully and completely to get an overview, 2) Read one direction at a time very carefully, 3) Think about and analyze each direction until you are sure you understand it exactly, and 4) Carry out the directions.⁶ Reading directions accurately is as important in math as it is in any subject because it requires intensive reading and attention to detail. Thus, teachers should stress the importance of understanding and following directions.

Third, a teacher can strengthen her students' background of understanding by relating new topics to topics previously studied. This "relation back" also will assure the teacher that the students fully understand the previous concepts. For example, assume that a student in Algebra who studied the properties of exponents in the previous chapter-- ie. Product of Powers: $x^m \cdot x^n = x^{m+n}$ -- is to learn how to multiply polynomials in the new chapter. The teacher should point out that in simplifying the polynomial $m^3n^2(2m^2 - 3mn + n^2)$ the students are applying the Product of Powers Rule which they learned in the previous chapter. Another example of developing a conceptual background would be to relate the addition, subtraction, multiplication and division of polynomials to similar processes dealing with whole numbers with which the students already are familiar.

Fourth, the teacher can make the student aware of reading methods appropriate for mathematics. The technique for reading mathematics material is entirely different from the traditional method of reading in which the reader often skims the material and focuses only upon the main idea in the material? The teacher should point out that mathematical material generally requires a slow, word by word method of reading and that it may be necessary to read the material several times for different purposes. Aukerman suggests that the students first should read at a moderate rate to get an overall view of the content and to answer the following questions: 1) "What is to be proved?", 2) "What is the unknown?", 3) "What vocabulary needs clarification?", 4) "What must I know to solve for the unknown?", and

5) "What information am I given?". The student then should reread the material at a slower rate (but not word by word) to note the relationships and the logical development of the problem.⁸

Three strategies which emphasize how to read and study a word problem are The Directed Reading Method, Word Problem Reading Reading Guide and PQ4R. (See Word Problems for discussion and examples. Some additional reading guides - not applied to word problems - are included in the following activities.)

A fifth suggestion for enhancing comprehension of mathematical material is to have the students "read with paper and pencil".⁹ This method does not refer to traditional note-taking techniques, however. Rather, the student uses paper and pencil to internalize, comprehend and recall information. For example, the student may draw a diagram, labeling the given information, as he reads a problem to aid in his thinking processes.

The final technique to enhance mathematical reading skills is to familiarize the students with the mathematics textbook. (See Activities 3 and 8).¹⁰ For students to get the most benefit from their mathematics textbooks, they need to learn what information is available to them in the textbook and to become familiar with the format of the book. This familiarity can aid students in locating information and thus increase their understanding of their textbook.

1.

Reading Guide

Chapter 8 Factors and Exponents

Choose the best answer(s) for the terms below. There may be more than one.

1. Coefficients of polynomials
 - a) the numbers in small type
 - b) must be an integer
 - c) the numbers to the right of the variable
2. Variable
 - a) the only letter in a math sentence
 - b) something that can have different values
 - c) a number which cannot be found
3. Perfect number
 - a) an integer perfectly formed
 - b) any real number used correctly
 - c) an integer equal to the sum of its proper divisors
4. Proper Factor
 - a) all factors of an integer which are also integers
 - b) factors of a composite integer
 - c) any real number that is a factor
5. Prime number
 - a) a number which is only divisible by 1 and itself
 - b) always an odd number
 - c) each factor of an integer after complete factorization

Miller, p. 390

2.

Study Guide

Pythagorean Theorem

Directions: The following passage was taken directly from your text. Every fifth word has been deleted. Without looking at your text see how many sensible words you can replace. Hint: Sometimes the word that makes the most sense is a very simple word, rather than a "mathematics" word. You may be surprised at how many words you can supply that make the completed passage sensible reading. Feel free to share your ideas with your working partner, and be sure to ask him for ideas. When you have completed as much of the "message" as you can, refer to page 242 in your text, find out how well you did, and complete the message so the author would approve of it.

The theorem to be proved in this section is one of the best known and most useful theorems in all mathematics. It states an _____ relationship between the lengths _____ the legs and the _____ of the hypotenuse of _____ right triangle. Most of _____ are already familiar with _____ relationship, perhaps knowing it _____ such names as the "_____ Triangle Principle," the "Pythagorean _____," or the "Rule of _____."

In any right triangle _____ square of the length _____ the hypotenuse is equal _____ the sum of the _____ of the lengths of _____ legs.

3.

Scavenger Hunt

The teacher could make a list of locational type questions and conduct a class session where students look for information. The class can be divided into two teams. One question is asked at a time and the students look for the answer. When an answer is found by a team, it is shown to the entire class and that team receives a point.

Suggested questions:

- 1) On what pages did you find information about absolute value?
- 2) How many pages are in chapter two?
- 3) What is the title of chapter twelve?
- 4) How many subtitles are in chapter one?
- 5) Who is the author of the book?
- 6) What is the copyright date of your book?
- 7) What is the fifth word in the glossary?
- 8) What information can you find about the cover?
- 9) On what page is the chapter summary for chapter three?
- 10) Give the name of the publisher.
- 11) Where is the preface located?
- 12) Locate the tables in your book and give the uses of each.
- 13) Are answers to all problems given on the answer pages?
- 14) How many pages of answers are there?

4.

Reading Rates

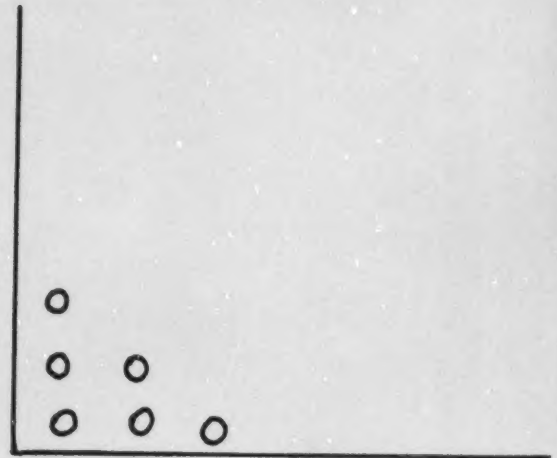
Sample Activity: When the students have read a passage, involve them in a contest to find all the mathematics words they can find in a certain time limit. Then give them a time limit to find additional information. Finally, have them close their books and see how much they remember. As the students orally recall information, the teacher writes this information on the board. Now, have the students go back to their books and check to see how much information was not remembered from the skimming activity.

Sample Activity: Assign a page in a mathematics book to be scanned by the students. After they scan it, have them summarize it. When this is finished, have them compare what they comprehended when they scanned to what they comprehended when they read carefully.

Symbolic Graphs

Some graphs use a symbol that represents a larger number. Pick your own axis for whatever labeling you want, but you must pick an axis for the names John, Pat, and Henry, in that order, either left to right or top to bottom.

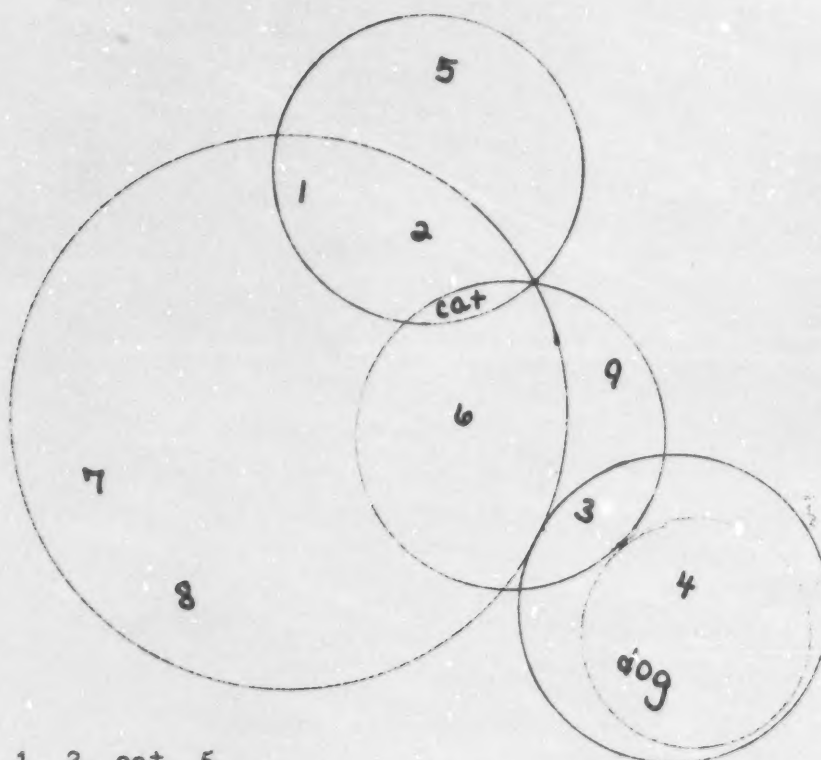
- 1) Who has the most snowballs if names are labeled on the vertical?
- 2) Who has the most snowballs if names are labeled on the horizontal?
- 3) Who has three snowballs if names are labeled on the vertical?



KEY: ○ = 3 snowballs

6.

Interpreting Venn Diagrams



Set A = 1, 2, cat, 5

Set B = 3, 4, dog

Set C = 1, 2, cat, 7, 8

Set D = 4, dog

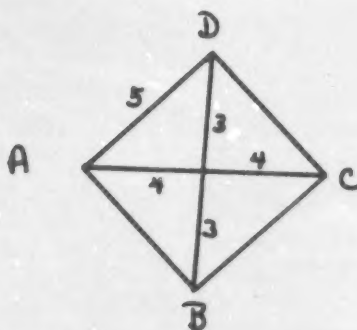
- 1) Trace the curve around Set A.
- 2) Trace the curve around Set C.
- 3) Does Set B contain any of Set A?
- 4) What set is totally encircled by another set?
- 5) Set E is drawn but not listed. List its elements.

7.

Reading With Paper and Pencil

Sample activity: Use passages such as the following to alert students to the importance of using paper and pencil when reading.

The converse of the Pythagorean Theorem states that if the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle. For example, a triangle with the sides of lengths 3, 4, and 5 is a right triangle, while a triangle with sides of lengths 2, 2, and 4 is not a right triangle. Using this theorem, one can show that quadrilateral ABCD is a rhombus.



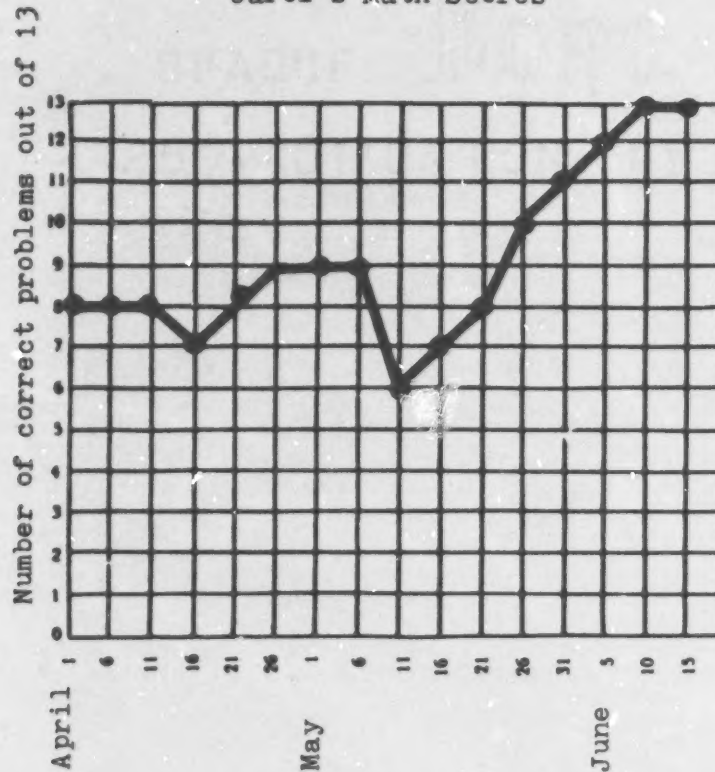
8.

Using Math Text

If students find a mathematical word they do not know, have them check the page to see if the definition is given. If it is not, then have them check the preceding pages and the glossary. Have the students complete the following table using their textbooks.

Word	Page on Which Definition was found	Method Used for Presentation of Definition

Carol's Math Scores



Directions: From the above graph, answer the following questions:

- 1) What is the purpose of the graph?
- 2) On what date did Carol get the least number of problems correct?
- 3) On what date did Carol get 12 problems correct?
- 4) On what date did Carol's score double a previous score?

10.

Interpreting Graphic Materials

Sample Activity: Have students make their own graphs. Small groups can graph different characteristics of the class. The following characteristics may be used:

- | | |
|------------------|---------------------------|
| -student heights | -types of books liked |
| -student weights | -types of movies liked |
| -color of eyes | -favorite school subjects |

Sample Activity: Give the students a graph and ask specific questions about it.

For example, the students may be given a bar graph which shows the number of points scored by ten different basketball players. The following questions might be asked:

Underline the statements which are true:

Denise made more points than Elijah.

Kyle made fewer points than Carlos.

Three of the players made 14 points each.

Betsy made fewer points than Bernard.

Roberta made more points than anyone.

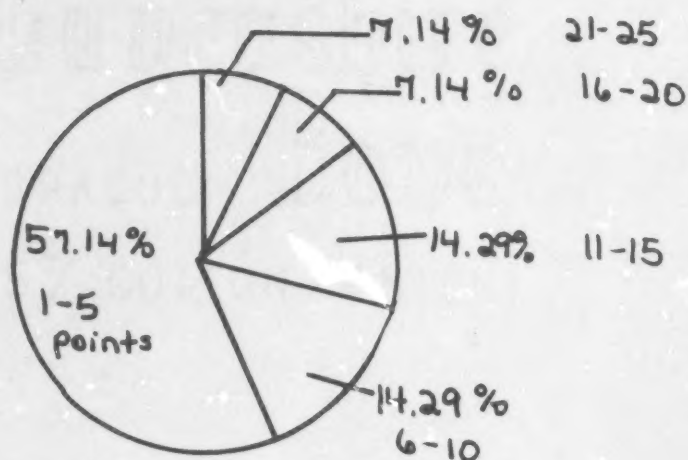
There are 12 players shown on the graph.

Carol made 17 points.

It is possible to show $7\frac{1}{2}$ points on the graph.

11.

Distribution of the Fourteen Basketball Players on a Squad
Whose Season's Average Scores Per Game Fell in Certain Ranges



1. How many players averaged between one and five points per game for the season?
2. What percentage of the team averaged eleven to fifteen points?
3. How many students averaged in "double figures"?
4. How many students averaged nine points or less?
5. Classify by categories how you spend your money; estimate the percentage you spend for each category; construct a pie graph to depict the results. Compare your results with those of someone who used the same categories.

Hafner, p. 272

C. Annotations

Aaron, I. E. "Reading in Mathematics," Journal of Reading Vol. 8 (May, 1965): 391-395.

Aaron identified five areas of responsibility for the mathematics teacher in teaching mathematics. They include: 1) mathematical vocabulary, 2) concept background necessary for understanding ideas presented in mathematics materials, 3) ability to select skills and rates appropriate for materials being read, 4) proficiency in special reading tasks of mathematics--reading word problems, equations, charts, graphs and tables and 5) skill in interpreting mathematical symbols and abbreviations. The math teacher has the responsibility for teaching special reading skills necessary for reading mathematics effectively.

Aukerman, Robert C. "Reading in the Mathematics Classroom," Reading in The Secondary School Classroom. New York: McGraw-Hill Book Company, 1972, pp. 189-232.

Aukerman advocated teaching students how to read the mathematics textbook to improve achievement in mathematics. Reading skills identified and discussed were: ability to pick out main ideas, mastering vocabulary and symbols, understanding the grammar of mathematics, following directions, reading mathematical graphs, charts and tables and solving verbal problems. Seven suggestions for improving comprehension in mathematics reading were presented.

Call, R. and Wiggin, N. "Reading and Mathematics," The Mathematics Teacher Vol. 59 (February, 1966): 149-157.

The researchers investigated whether there was a correlation between a student's ability to solve word problems in second year Algebra and the presence or absence of special reading instruction. Two groups were taught the same unit in second year Algebra. An experienced mathematics teacher taught the control group. An English teacher with training in reading and no training in mathematics taught the experimental group. The primary difference in the teaching method of the two groups was that the experimental group was taught to obtain the meaning from the words and to translate them into mathematical sentences. The authors concluded: "If by teaching reading, instead of mathematics, we can get better results, it seems reasonable to infer that the competent mathematics teacher might get considerably better results if her were trained to teach reading of the kind encountered in mathematics."

Clary, Linda M. The Why and a Little How -- Teaching Reading in the Content Areas ERIC Document Reproduction Service, ED 176 231, 1974.

Clary believed that content teachers must first understand the reasons why students have trouble reading content materials before they can devise methods of helping their students. The primary reading problems in the content areas are student competence, content materials, curriculum pressures, and teacher education.

Clary prescribed the following possible approaches to modify content area reading: the language experience approach and the individualized reading approach. Clary believed that the teacher who uses both also demonstrates an insight into why students have difficulty reading and a willingness to employ strategies to help solve the problems.

Dolgin, Ann B. "Improvement of Mathematical Learning Through Reading Instruction," The High School Journal Vol. 61 (November, 1977): 59-69.

Dolgin stated that "content area teachers must recognize their responsibility to teach those necessary reading skills specifically related to their subject area." Necessary skills she discussed include vocabulary and comprehension. Comprehension skills should be taught at three levels: literal, inferential, and critical. An example of a comprehension lesson was presented. Dolgin suggested the use of an informal diagnostic survey, her own, to determine how well a reader understands a text, and the appropriateness of certain materials.

Earle, Richard A. Teaching Reading And Mathematics. Newark, Delaware: International Reading Association, 1976.

Earle presented strategies, techniques and ideas to assist mathematics teachers in teaching reading in the mathematics classroom. In addition, he presented suggestions and examples of implementing these strategies.

Earle's model of mathematical reading was based on Thomas Barrett's definition of reading. He described four heirarchical levels of reading: 1) perceiving symbols, 2) attaching literal meaning, 3) analyzing relationships and 4) solving word problems. This booklet is an excellent resource for the mathematics teacher because it is practical and useable in the mathematics classroom.

Florida State Department of Education, "Comprehension," and "Reading Skills," Reading the Language of Mathematics ERIC Document Reproduction Service, ED 134 972, 1975, pp. 19-27, 34-42.

The authors classified skills and abilities involved in getting meaning from a language into literal and interpretative comprehension. Understanding the direct meaning of a word, phrase or sentence is literal comprehension. Understanding the implied meanings is interpretative comprehension. Specific competencies believed necessary in understanding mathematics are: 1) students need to understand the main ideas and important details, 2) they need to be able to follow directions, 3) students need to be able to interpret graphic materials, and 4) they need to be able to understand mathematical sentences. The authors also discussed study techniques such as flexible reading rate, reading with paper and pencil, using book parts, and using a mathematics dictionary. Numerous activities in each of the competencies discussed were provided for both secondary and elementary mathematics students.

Georgia State Department of Education, Reading Mathematics ERIC Document Reproduction Service, ED 105 407, 1975.

In this pamphlet the authors discussed two processes involved in reading mathematics: eye movements and relating the appropriate word or symbol to the mathematical idea. Seven eye movements, such as forward, diagonal and bottom to top, were identified and examples of each were given. The authors pointed out many kinds of words can be found in the language of mathematics. Included are mathematical words, words with several different meanings, and words of everyday usage. Often, preconceived meanings of words need to be overcome before mathematical meanings can be substituted. Many examples of each of these kinds of words were presented.

Hafner, Lawrence E. "Reading Mathematics-Strategies for Improving the Reading of Mathematics," Developmental Reading In Middle and Secondary Schools: Foundations, Strategies, and Skills for Teaching. New York: MacMillan Publishing, Co., 1977.

Because the mathematics teacher is to help students think well in mathematical situations and reading is a thinking process, Hafner contended the mathematics teacher should help students read mathematical material. Hafner presented reading improvement strategies designed to help students read mathematical vocabulary and symbols, relate operations to corresponding mathematical symbols, and translate written formulas into mathematical symbols. He presented an example of a Directed Reading

Lesson adapted for mathematics. This chapter is an excellent source of sample strategies and activities useful for improving reading in the mathematics classroom.

Henrichs, Margaret, and Sisson, Tom, "Mathematics and the Reading Process: A Practical Application of Theory," The Mathematics Teacher Vol. 73 (April, 1980): 253-257.

Henrichs and Sisson reported on the efforts of mathematics teachers at West Junior High School in Columbia, Missouri to incorporate reading into the content areas. For three years they participated in a Title IV-C project, "Reading Instruction in the Content Areas." The program was designed for the improvement of reading and study skills for the students. In-service training for teachers was provided in content-area reading and study skills. Teachers learned to determine reading ability; to determine approximate reading level of their texts; and they learned content-area reading skills: vocabulary, comprehension, and study skills. Math teachers then defined and described reading and study skills for the subjects and they devised examples of activities to implement these skills.

Miller, Wilma H. "Individualizing Reading Instruction in Mathematics," Teaching Reading in the Secondary School Springfield, Illinois: Charles C. Thomas, 1974, pp. 380-400.

In this chapter Miller attempted to enable mathematics teachers to provide for individual reading differences

more effectively. He included the following materials and methods: informal reading tests, SQ3R, reading and reasoning guides, teacher-formulated or student formulated simplified verbal problems, the language experience approach, supplementary reading materials, exercises in vocabulary development and various audiovisual aids. The mathematics teacher will find this chapter useful because in addition to presenting strategies and methods, the authors included concrete examples of those methods.

Piercey, Dorothy. Reading Activities In Content Areas, An Ideabook For Middle and Secondary Schools. Boston, Mass.: Allyn and Bacon, Inc., 1976.

Piercey presented numerous activities which attempted to teach students the significance of mathematics to the world outside the classroom while promoting mathematical reading skills of the students. The activities applied such skills as reading and interpreting equations and graphic material, problem solving, estimating, evaluating and making judgments, recognizing relevant data, and discovering relationships. The activities were categorized according to subject areas - including general math, algebra, geometry and math in general. Included in each activity was the recommended grade level, objective of the activity, the teacher and student preparation and the procedure.

Robinson, H. Alan, "Mathematics," Teaching Reading and Study Strategies The Content Areas Boston, Mass.: Allyn and Bacon, Inc., 1975, pp. 159-188.

Robinson believed that strategies for reading and studying mathematics must concentrate on patterns of presenting concepts and principles as well as on solving mathematical problems. He presented methods and techniques providing guidance for students to cope with the heavy and specialized vocabulary load, the integration of symbol systems, the variety of graphics and the major patterns of writing in mathematical textbooks. The author delineated six steps for approaching the study of concepts in mathematics and six steps for understanding the mathematical principles and learning how to apply it.

Santeusano, Richard P. A Practical Approach To Content Area Reading Reading, Mass.: Addison-Wesley Publishing Company, 1983.

Santeusano provided specific, practical and effective techniques for incorporating the teaching of reading in any subject area. The mathematics teacher will find these techniques and suggestions applicable in teaching reading in the mathematics classroom. He did not offer a "cookbook" of reading recipes but presented suggestions and techniques with some theory and related research. This is a good book for the educator who desires to learn more about developing self-directed study guides, identifying student needs and applying readability formulas, for example.

Shephard, David L., "Applying Reading Skills To Mathematics," Comprehensive High School Reading Methods. Third Edition Columbus, Ohio; Charles E. Merrill Publishing Company, 1982, pp. 305-328.

Shephard presented an excellent chapter on teaching reading skills in the mathematics classroom. He discussed the nature of mathematical writing, nine reading skills that apply to mathematics, and inadequate readers in the mathematics class. Nine specific suggestions for overcoming difficulties in reading problems were presented. Included among them were: 1) have many exercises in vocabulary study, 2) have the students visualize the problem situation, 3) have the students read many different types of problems, and 4) have the students in class discussion talk through the problem to determine everything the problem does and why.

Texas Education Agency, "Comprehension," and "Study Techniques," Learning Partners: Reading and Mathematics ERIC Document Reproduction Service, ED 179 422, 1979, pp. 16-22, 28-34.

This pamphlet appeared to be based on the same outline as the Florida State Department of Education, 1975. Likewise, these authors contended literal and interpretative comprehension were important in understanding mathematics. They pointed out that sometimes an operation is implied rather than stated directly in a mathematics problem. The authors suggested that when students are reading graphic materials, they should first survey the material to determine what it is

about, how it is organized and what information is supplied. Teachers can stimulate interest in voluntary mathematics-- related reading material-- in order to encourage better reading of mathematics. Books were suggested by the authors for voluntary reading. Many activities to encourage mathematical reading skills were presented.

Thomas, Ellen Lamar and Robinson, H. Alan, "Mathematics," Improving Reading In Every Class A Sourcebook For Teachers. Boston: Allyn and Bacon, Inc., 1972, pp. 277-325.

The authors believed the mathematics teacher should provide her students with appropriate reading guidance to aid success in their present math course and to insure success in all the students' advanced mathematics work. Three topics were discussed in this chapter. First, comprehensive guide sheets written to the student were presented. These guide sheets discussed why reading in mathematics requires a "new look" and presented approaches and techniques for students to use when reading mathematics assignments. Second, Thomas and Robinson presented helps for teachers in improving mathematics reading. Ideas such as pre-teaching vocabulary words, using standardized reading scores to evaluate students, conducting "How to Read Your Textbook" sessions, and creating readers in mathematics for life were discussed. Third, the authors focused on mathematical word problems. They offered several suggestions

for dealing with word problems. First, a step by step procedure was encouraged. Second, diagrams and estimations as well as verbal checks of the problems were recommended. A third idea was to try reading problems orally and also to have students bring in "live" problems.

Toback, A. S., "Secondary Teachers Can Teach Reading," The Clearing House Vol. 49 (1976): 248-249.

Toback advocated secondary teachers dispell two myths about reading at the secondary level: 1) Reading should take place only in the elementary grades and 2) Only those students who have reading problems need help. He presented a series of ten steps for teachers to help their students become better readers in the content areas. Mathematics teachers will find these suggestions very useful to improve effective reading in mathematics.

FOOTNOTES

¹I.E. Aaron, "Reading In Mathematics," Journal of Reading Vol. 8 (May, 1965); 391; Collier and Redmond, "Are You Teaching Kids To Read Mathematics?", p. 805; Dolgin, "Improvement of Mathematical Learning Through Reading Instruction," p. 59; Earle, Teaching Reading and Mathematics, p. 85; Wilma H. Miller, "Individualizing Reading Instruction In Mathematics," Teaching Reading In The Secondary School (Springfield, Illinois: Charles C. Thomas, 1974), p. 383; Shephard, "Applying Reading Skills to Mathematics," p. 307; Thomas and Robinson, "Mathematics," p. 277.

²Aaron, "Reading In Mathematics," p. 391; Shephard, "Applying Reading Skills To Mathematics, p. 307.

³Texas Education Agency, "Comprehension," Learning Partners: Reading and Mathematics, p. 19.

⁴Shephard, "Applying Reading Skills To Mathematics," p. 311.

⁵Ibid., p. 318

⁶Ibid., p. 322.

⁷Aukerman, "Reading In the Mathematics Classroom", p. 189; Hafner, "Reading Mathematics, Strategies for Improving the Reading of Mathematics," p. 258; Holliander, Reading In The Special Language of Mathematics, p. 5; Thomas and Robinson, "Mathematics," p. 277; Krulik, "To Read or Not To Read, That Is The Question!", p. 248.

⁸Aukerman, "Reading In The Mathematics Classroom," p. 194.

⁹Florida State Department of Education, "Study Techniques," Reading the Language of Mathematics, p. 31.

CHAPTER VI

READABILITY OF MATHEMATICS MATERIALS

A. Discussion of the Reading Problem

Most secondary mathematics teachers consider the mathematics textbook to be an integral part of the instruction they provide their students.¹ According to Santeusanio, however, "providing the student with a tool he cannot use is really providing him with very little."² Moreover, Elliott and Wiles believe that the mathematics texts currently being used are not as helpful as the dependency on them would seem to warrant. Elliott and Wiles attribute the problem to the decline in students' reading skills and the mathematics teachers' lack of training to teach reading skills. In their study, they used a Cloze test and found that over one-fourth of the mathematics teachers they evaluated were unable to handle a textbook which was widely used at the eighth grade level. They concluded that it is difficult for trained mathematics teachers to independently completely process a text with mathematics concepts embedded in the reading.³ Dolgin attributes readability problems to the dual language system (symbols and written language) found in mathematics textbooks and also the varied student reading ranges found in the classroom.⁴

Problems arise when assessing the readability of mathematics material. Readability formulas typically used, such as

Dale-Chall, Spache, and Flesch methods, depend on the length of words and sentences, and number of unfamiliar words. These formulas usually underestimate the difficulty of reading in mathematics because they do not consider concept difficulty and technical language.⁵ Kulm points out that the variables that affect the reading difficulty of ordinary English are different from those that make mathematics difficult to read.⁶ Likewise, according to Kane these readability formulas are inappropriate for mathematics because 1) they ignore critical reading segments that are not presented in English in symbols, and 2) the material presented in English is filled with words and phrases from mathematical jargon.⁷

B. Strategies, Activities and Exercises for Overcoming Mathematics Reading Difficulties

There are several methods which may be used to estimate whether students can read their textbooks. Mater and Kane have proposed one such method for assessing the readability of mathematics material by using a modified Cloze procedure.⁸ The Cloze procedure presents the student with a text of approximately 275 words. The first sentence is given in its entirety. Every fifth, eighth, or tenth word is omitted. The student fills in the blank spaces with the word he thinks was deleted. A percentage score is computed to determine whether the material is at the student's independent (58% and above), instructional (40-56%), or frustration (0-38%) level.⁹ This procedure indicates whether the given material is useable for the readers to whom it is presented. Mater and Kane found the Cloze scores to be

highly correlated with comprehension tests for the same passages.¹⁰ Kulm devised a similar method to assess the readability of Algebra textbooks which used variables such as percentage of mathematical symbols, average sentence length, percentage of mathematical vocabulary words and percentage of reader-directed sentences.¹¹ He concluded that symbol difficulty far outweighs vocabulary difficulty. The procedures used by Kulm, Hater and Kane both take into account mathematical expressions and symbols, unlike standard readability formulas.

A second method to determine the reading efficiency of mathematics students is the Group Reading Inventory (GRI).¹² Like the Cloze procedure, the GRI is based on classroom text material. The teacher analyzes the text to determine those skills the students will use most often as they read the texts. Test items are constructed from textbook passages that measure student proficiency in those skills. In addition to measuring skills, the GRI can show the student the proper use of the mathematics textbook. A checklist of each skill is compiled for each student. Questions such as using book parts, understanding symbols and vocabulary, and following directions may be included. Shephard suggests that seven skills should be included in a mathematics inventory: 1) reading verbal problems and stating them in one's own words, 2) adjusting one's reading to the requirements of the problem, 3) translating words into symbols, 4) knowing the meaning of symbols, 5) understanding vocabulary, 6) noting the relationship in formulas and equations, and

7) obtaining information from charts, tables and graphs.¹³ (A sample form of inventory is included in the following activities).

Assessing a student's ability to read a mathematics text is not easy. A teacher may want to use some form of a Cloze test in order to determine the general ability of the student to read the mathematics book of that particular class. Group Reading Inventories are useful for those teachers who want to determine strengths and weaknesses of their students in specific skills. As Earle points out, some teachers may find that they learn more about their students' reading success from their daily association with them in the mathematics classroom than from formal tests or inventories.¹⁴ In assessing the readability of mathematical material the teacher must keep in mind that the students' background experience and mathematical knowledge have a considerable influence on their understanding of mathematical material.

1.

Diagnostic Survey

The following procedure is a means to determine how well the reader may understand a textbook but teachers should modify it to fit individual situations.

1. Select a passage from an assigned mathematics textbook totalling 100 to 150 words. You may use a single topic, or combine several topics, but make sure each topic will stand on its own merit of independent ideas. If a passage refers to a figure, graph, or chart be sure to include it. Determining words in English prose presents no difficulty but when symbols are involved, personal judgment should be made.
2. Pre-read the selection and write five comprehension questions. Four may include the literal level of comprehension or what the author expresses directly. One question may deal with vocabulary or the meaning of a specific word found in the selection. For example:
 - a) Vocabulary: What is a trapezoid?
 - b) Comprehension: How is the area of a trapezoid determined?
3. Type the reading selection. Then type the questions on a separate page. Make ditto copies of each.
4. Administer to an individual student, small group, or the entire class. This procedure is designed to determine if a student or group comprehends a textbook or supplementary materials.
5. Inform your student(s) that this exercise is not a test but a means to help them improve their understanding of the textbook.
6. Set the purpose for reading by stating the topic and provide directions.

7. Have students read the material silently. Upon individual completion collect the reading selection and distribute the questions. Allow as much time as you think is appropriate.

8. Collect the materials and score them. Partial credit may be given for an answer. In order for material to be considered appropriate at the instructional level Pikulski (1974) has provided some guidelines for determining reading levels based in part upon having students read a passage and then answer comprehension questions about the passage. If a student receives, 75 to 90 percent then he is reading on an "instructional" level. That means that a student can comprehend materials if he is given assistance by the teacher. At this level the teacher ought to provide a general overview of what is to be read; provide some guiding questions to be followed by a classroom discussion after the material has been read. Any score below 75 percent indicates a "frustration" reading level which means that a student is unable to handle assigned reading materials. Word recognition and comprehension are so difficult that a student may show signs of visible anxiety. A score over 90 percent may represent the "independent" reading level. This means that a student has no difficulty in word recognition or comprehension and can thereby read materials without teacher assistance. A general guideline to observe is to use materials which are at least on an "instructional" level recognizing that the teacher must direct and guide students with their reading assignments.

Dolgin, p. 67-68

Sample Form of Inventory

Four to five questions per skill

Restating verbal problems

Use typical verbal problems for the subject. Have students read the problems and write the situations they posed. Questions may ask also for what is given and what is asked for (detailed reading). Questions which ask students to note the basic mathematical processes may also be used (interpretative reading).

Adjusting one's reading

Students may be asked to state how they would read specific problems, what steps they would use, whether they would read rapidly or slowly, or how they think the reading of mathematical problems differs from story-type reading.

Translating words to symbols

Ask students to underline the words and phrases in a problem that should be converted into mathematical notations. The appropriate symbol may be required of specific words or phrases.

Knowing symbol meanings

Give specific symbols and have the students write the meanings of each.

Understanding vocabulary

Include questions of both generalized and specialized vocabularies:

- a) Words representing ideas of quantity (big, bigger, long, wide, etc.)
- b) Words used to represent number figures.
- c) Words used to represent number processes.
- d) Words used to represent kinds of measurement.
- e) General terms (plus, minus, more than, increased by).
- f) Specialized technical words (radius, isosceles).

Noting relationship in formulae and equations

Use a problem and have the students express the relationship in the form of a formula or equation. Also, an equation may be used expressing the relationship in words.

Obtaining information from charts, tables and graphs

Reproduce charts, tables, and graphs and ask questions requiring students to use the graphic representations. Present a problem and have students make a graphic representation of it.

Sample Textbook Survey

PART I

Directions: Using your textbooks, answer the following questions.

1. How many chapters are in the book? _____
 2. Who are the authors of *Using Algebra*? _____

 3. A. For an angle of 66° , what is the sine? _____ cosine? _____
tangent? _____
B. On what page did you find this information? _____
 4. A. What is the square root of 145? _____
B. On what page did you find this information? _____
 5. In which chapter will you learn to factor polynomials? _____
 6. A. What is the rule for multiplying a positive and a negative number? _____
B. Where did you find it in the text? _____
 7. In which chapter will you learn about Trigonometry? _____
 8. On what page will you look in Chapter 6 to see if you are ready to take a chapter test? _____
 9. Although each chapter discusses a different topic, there are some things that all chapters have in common. What are they? _____

 10. Throughout the book are puzzles, games and brain teasers. What page indicates where these "Special Topics" are found? _____
 11. On what page will you find the following symbols introduced?
 $\cap, \cup$ _____ $\sqrt{\quad}$ _____
 $f: x \rightarrow y$ _____ $0.\bar{0}$ _____
 12. Are the answers to C.e problems in the book? _____ If so, which problems are answered and on what pages will you find them? _____

 13. What makes the definitions in the book stand out on the page? _____

 14. Using the Table of Contents, list five words or terms that you are not familiar with. List sections in which that phrase or word is listed.
- | Word | Section |
|-----------|---------|
| (1) _____ | _____ |
| (2) _____ | _____ |
| (3) _____ | _____ |
| (4) _____ | _____ |
| (5) _____ | _____ |

15. Write a definition for each of the five words or terms that you listed above.

- (1) _____
 (2) _____
 (3) _____
 (4) _____
 (5) _____

16. What is the sum of $\frac{11}{24} + \frac{7}{30}$? _____

17. On what page are examples of addition of fractions found? _____

PART II.

Directions: Using the page numbers listed with each question, answer the following questions.

1. Listed alphabetically below are some operations of real numbers. Give an example of each with its inverse. (p. 188)

- | | |
|-------------|-------------------|
| A. Addition | C. Multiplication |
| B. Division | D. Subtraction |

2. There are two methods to multiply polynomials. What are they? (p. 216)

- (1) _____
 (2) _____

3. On a coordinate plane (p. 284), which is the

- x — axis? _____
 y — axis? _____

4. When an equation (p. 282) is in the form $y = mx + b$,

- what does m represent? _____
 what does b represent? _____

5. If a and b are any real numbers, $b \neq 0$, and n is a positive integer, then what does

$\left(\frac{a}{b}\right)^n$ equal? (p. 342) _____

6. There are two types of problems involving time. What are they? (p. 394)

- (1) _____
 (2) _____

7. Page 64 discusses the distributive property. Let a , b , and c be any real numbers, and complete the statement

$a \cdot (b + c) =$ _____ (p. 64)

8. Let a , b , and c be any real numbers.

If $a = b$, then $a + c = b + c$. This is an example of what property? (p. 82) _____

9. What is a prime number? (p. 153) _____

10. What do the letters in FOIL stand for? (p. 219)

- | | |
|-----------|-----------|
| F — _____ | I — _____ |
| O — _____ | L — _____ |

Elliott, Peggy Gordon, and Wiles, Clyde A. "The Print Is Part Of The Problem," School Science and Mathematics Vol. 80 (January, 1980): 37-42.

Two problems with readability of mathematics textbooks exist. First, mathematics students have problems reading their textbooks and require help to do so. Second, often mathematics teachers are not trained to teach the reading operations necessary for handling the textbooks. As a result teaching and learning which is dependent upon the text comes to a halt. The authors investigated "the print problems" for readers in current textbooks in mathematics by administering the Cloze test to mathematics teachers. One-fourth of the teachers were unable to deal easily with the material from an eighth grade textbook. Elliott and Wiles suggested several possible solutions such as to train math teachers and to improve the reading in the materials themselves.

Hater, Mary A. and Kane, Robert B., "The Cloze Procedure As A Measure of Mathematical English," Journal for Research In Mathematics Education Vol. 6 (March, 1975): 121-127.

The authors found support for their hypothesis that Cloze tests adapted for mathematical English passages are valid predictors of reading and comprehension for grades 7-10. In addition to administering a Cloze test the researcher administered a multiple-choice comprehension test on the same passage. They found: 1) no evidence that responding to a Cloze test affected a

later performance on the comprehension test on the same text material, 2) high correlations between the Cloze and comprehension test results for the same passages. The authors suggested further research using more passages in the study to verify that Cloze tests are valid predictors for mathematical English.

Kane, Robert B., "The Readability of Mathematics Textbooks Revisited," The Mathematics Teacher Vol. 5 (November, 1970): 579-582.

Kane differentiates between mathematical english and ordinary english. He feels they are different enough that readers require different skills and knowledge for reading comprehension. Kane contended that readability formulas such as Dale-Chall, Flesch and Spache are inappropriate for rating mathemtaics textbooks for two reasons: 1) large critical reading segments must be ignored because they are presented in symbolic systems, and 2) English sentences are filled with words and phrases from mathematical jargon. The author pointed out readability formulas for mathematics texts are being developed.

Kulm, Gerald, "Sources of Reading Difficulty in Elementary Algebra Textbooks," The Mathematics Teacher Vol. 66 (November, 1973): 649-652.

Kulm investigated whether it is necessary to sample separately the prose and the symbolic material in Algebra textbooks and then apply separate formulas for readability. Two types of written material were identified in

Algebra textbooks: explanatory and illustrative. Findings indicated the greater the percentage of mathematical symbols, the more difficult the material--- either illustrative or explanatory---is to read. Kulm concluded that vocabulary is important in reading mathematics but the difficult symbolism is a much greater factor. He implored the Algebra teacher, in particular, to be aware of the difficulty symbols present to students in reading Algebra. He also suggested teachers enable students to study and practice to develop facility with symbolism in mathematics.

Lees, Fred, "Mathematics and Reading," Journal of Reading Vol. 19 No. 8 (May, 1976): 621-626.

Lees suggested that math teachers at all levels can improve their students' learning of mathematics by improving the ability of the student in reading the material. Use of reading techniques and materials to help students learn mathematics by diagnosing their needs, constructing and utilizing comprehension aids, and assigning homework in a realistic manner were among Lees' suggestions. First, teachers must diagnose needs by 1) readability scales: Fry, Flesch, or Dale-Chall; 2) Cloze test; 3) an informal reading inventory; 4) standardized reading ability tests. Then by using the results of the diagnosis the teacher must construct and provide aids (charts, diagrams, hands-on-experiences) to help the student. Finally, homework should have

time devoted to the how, what, why and when of the assignment. Students benefit greatly from knowing how to approach the assignment, what to look for, why they are doing it and when it is due.

McCabe, Patrick P. The Effect Upon Comprehension of Mathematics Materials Repatterned On The Basis of Oral Language
ERIC Document Reproduction Service, ED 140 275, May, 1977.

McCabe examined the effect of rewritten mathematical material upon comprehension of elementary and junior-high school students. The material was rewritten on the basis of the students' oral language patterns. In her study the author found rewriting the material had a greater effect on the comprehension of the expository portion than on the narrative portion of the language. McCabe concluded that the difference in syntactic pattern of the two passages lead to the difference in comprehension. She recommended teachers should first become good listeners and then should include in their oral or written exposition as many of those linguistic structures employed by their students as possible.

Sirois, Herman A., and Davis, Robert L. Readability vs. Read-Ability: Instructional Design for Reading and the Content Area
ERIC Document Reproduction Service, ED 230 894, 1983.

Sirois and Davis distinguished between "readability:" the difficulty of printed material and "read-ability:" students' ability to understand printed material. They contended that often students can decipher combinations

of letters, but have little comprehension of the material. The booklet deals with the problems of 1) identifying readability of texts, 2) identifying read-ability of students, 3) matching the conceptual levels of the texts with the conceptual levels of students, and 4) identifying and implementing adaptive teaching strategies that compensate for situations where student and text conceptual levels do not match.

Smith, Cyrus F., and Kepner, Henry S., Jr., "The Readability of Mathematics Materials," Reading in the Mathematics Classroom ERIC Document Reproduction Service, ED 207 836, 1981, pp. 42-51.

The authors discussed methods frequently used in assessing the readability of texts. One component of the discussion was using readability formulas to evaluate mathematics textbooks. Other tools useful for determining textbook readability included the Cloze procedure, eighteen items cited by Krause that should be considered in evaluating a textbook, and the textbook survey. The current trend is to use mathematics textbooks with minimal reading, but the authors questioned that technique because it keeps the student from becoming an independent learner of mathematics.

FOOTNOTES

¹Perry Gordon Elliott and Clyde A. Wiles, "The Print Is Part of the Problem," School Science and Mathematics Vol. 80 (January, 1980): 37.

²Richard P. Santeusanio, A Practical Approach To Content Area Reading (Reading, Mass.: Addison-Wesley Publishing Company, 1983), p. 275.

³Elliott and Wiles, "The Print Is Part of the Problem," p. 41.

⁴Dolgin, "Improvement of Mathematical Learning Through Reading Instruction," p. 59.

⁵Elliott and Wiles, "The Print Is Part of the Problem," p. 38.

⁶Kulm, "Sources of Reading Difficulty in Elementary Algebra Textbooks," p. 651.

⁷Robert B. Kane, "The Readability of Mathematics Textbooks Revisited," The Mathematics Teacher Vol. 5 (November, 1970): 579.

⁸Mary A. Hater and Robert B. Kane, "The Cloze Procedure As a Measure of Mathematical English," Journal for Research In Mathematics Education Vol. 6 (March, 1975): 122.

⁹Santeusanio, A Practical Approach To Content Area Reading, p. 277.

¹⁰Hater and Kane, "The Cloze Procedure As A Measure of Mathematical English," p. 125.

¹¹Kulm, "Sources of Reading Difficulty In Elementary Algebra Textbooks," p. 650.

¹²Santeusanio, A Practical Approach To Content Area Reading, p. 281.

¹³Shephard, Comprehensive High School Reading Methods, p. 161.

¹⁴Earle, Teaching Reading and Mathematics, p. 61.

CHAPTER VII

CONCLUSION

As stated repeatedly in this paper, the vast majority of researchers who have examined the relationship between reading skills in general and achievement in mathematics have concluded that there is a strong correlation between a student's reading level and his achievement in mathematics. More specifically, although some researchers disagree, as a student's mathematical reading ability improves, his achievement in mathematics also improves.

In this paper the author has highlighted five mathematics reading problem areas which researchers have identified, and attempted to provide guidance for secondary mathematics teachers who desire to maximize their students' performance in mathematics by improving their mathematics reading ability. The author believes that this is important because teachers need to recognize the relationship between mathematical reading ability and achievement in mathematics. Moreover, after recognizing this relationship, teachers need to have a convenient means of accessing sources that suggest methods for improving their students' reading skills. Therefore, each chapter was divided into: 1) a discussion of a mathematics reading problem with a focus on the opinions of researchers, 2) suggestions for

overcoming mathematics reading difficulties, and 3) annotated bibliography of sources to which teachers can refer for ideas. Thus, although the paper has not discussed all of the factors relating to mathematics reading problems, the author has attempted to provide a basic research tool for teachers to utilize at least as a starting point for discovering methods to improve their students' mathematics reading ability.

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